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THE PHYSICAL SIGNIFICANCE OF THE DYER'S SYSTEM OF COLOUR MATCHING

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ABSTRACT. The dyer's method of specifying colours in terms of " shade ", " strength " and " dullness " is described and compared with the monochromatic system of colorimetry. Experimental evidence indicates that shade is probably equivalent to dominant hue in all respects. Differences of shade expressed by the dyer in a quasi-quantitative manner are shown to be related to just-perceptible steps of hue as determined by Wright. Changes in strength and dullness both involve simultaneous changes in purity and lightness (brightness factor), but statistical treatment of the result of an examination of dulled patterns by trained colourists and non-colourists shows that the former are guided by the physical quality of purity and the latter by lightness in assessing strength.

§1. THE DYER'S PROCEDURE

THE dyer's problem usually consists in dyeing yarn or fabric to match a small pattern of another colour, sometimes of different material, and the technique employed is to take advantage of previous experience. In his pattern book, which consists of patterns from colours that he has dyed previously, that which is nearest to the required pattern is sorted out, bearing in mind the fastness properties required of the dyed material. If the records are sufficiently complete, and the material is identical with that which has been previously dyed, then the necessary quantities of dyes will be applied with, in most cases, complete success. On the other hand, if the dyer is matching on material which is different in weave or weight from any he has used before, and his range of patterns is incomplete, then he will proceed with more caution, approaching the desired pattern by stages. This is necessary, as it is technically undesirable to put more dye on the pattern than is required for matching, since it may be very difficult to remove it once the pattern has been "overshot". Proceeding in this way he will, therefore, compare the colours obtained with that of the pattern from time to time. At the first comparison it may be found that the dyeing differs from the pattern in several aspects. Firstly, it may differ in a way which the dyer knows can be overcome by placing more dye on the fibre. This difference is described by saying that the dyeing is *weaker*, other phrases used being *thinner*, *empty*, *lacking in colour*, *pale*, or *washy*. Secondly, the dyeing may differ in shade, being redder, yellower, greener, or bluer, depending upon its colour, and it may differ in a third respect (being greyer or blacker), and this

is known as *dullness*. (The opposite effect is referred to by the dyer as *brightness*.) Thus at any stage in dyeing the dyer may find that his dyeing differs from the pattern in three respects—strength, shade and dullness. The dyer's objective is to build up the dye on the material until he obtains an exact match for the pattern, and then the dyeing is equal to the pattern in all the above three respects.

The quality of a pattern which comprises *shade* is very similar to the *hue* of physical nomenclature, and the difference in hue between two patterns can be detected by an untrained observer. The remaining two variables are rather difficult to define exactly, and are frequently confused one with the other, disagreement often arising as to whether one pattern is duller than another but equal in strength, or whether it is merely stronger. If two samples of a dye are examined which are identical in all respects except that sample A contains 50 parts of pure dyestuff in 100 parts of the commercial quality, while sample B contains only 45 parts per 100 parts, this difference may be expressed by the statement that sample A is 10 % stronger than B. Apart from the economic aspect, the relative strength of a dye is of great importance in practical dyeing, for if a combination shade containing other dyes is being dyed, it is essential to know that 90 parts of A can be replaced by 100 parts of B. From this it might be concluded that strength is not a colorimetric property at all, but merely a matter of chemical analysis. This is not the case, however, for strength is nearly always assessed visually, and frequently dyestuffs which are not chemically identical are compared for strength. Visual estimation is essential, as it is sometimes found that samples containing the same quantity of dye by analysis do not give dyeings of equal depth owing to differences of affinity. Similarly, analysis is no indication of the relative tinctorial powers of two dyes of different chemical constitution. The dyer must rely, therefore, upon the visual appearance of patterns dyed with the dyes under test.

The practical determination of strength is carried out by dyeing both standard and sample in a range of concentrations and comparing the resultant patterns. To avoid complications, an idealized procedure is described in which a series of dyeings is prepared containing 0·85, 0·90, 0·95, 1·0, 1·05, 1·1 % of the standard dye (sample A). Simultaneously a dyeing of the second sample, B, is made at 1·0 % and this dyeing is compared with the A series. If, as already stated, the samples are identical in all respects except strength, the colourist will find that the 1·0 % dyeing of B appears visually identical with the 0·9 % dyeing of A, and will conclude that B is 10 % weaker than A but equal in all other respects.

In the particular case postulated above, the results obtained by visual examination agree with the chemical analysis, but in many cases this is not so. For instance, the absorption curve of another sample, C, may be less sharp than that of A, owing to the fact that C is of a different chemical constitution, or perhaps contains some dark-coloured impurity from the manufacturing process. In this case a dyeing of C will never be visually identical with a dyeing of A. This difference, which cannot be eliminated by varying the relative quantities of dyes, is known to the dyer as *dullness*, the antonym being *brightness*, as already stated above. From practical experience it is known that if two dyes differ in this respect they can be made equal by adding a small proportion of black dye

to the brighter sample, and this fact may serve as a definition of dullness. A pattern is duller than standard if it is necessary to add black to the standard to obtain perfect visual equality, and similarly a pattern is brighter than standard if black must be added to the pattern, with the necessary adjustment of strength.

Clearly, if two dyes differ in dullness it becomes a difficult matter to estimate the relative strength, and the values obtained by visual examination may no longer agree with the analytical data. Ideally, the colourist attempts to discount the effect of dullness mentally, but this is a very difficult operation, involving a personal element, and individual estimates of the strength of a dull pattern show considerable variation.

In spite of the difficulty of estimating these three variables visually, the colourist is usually able to make a qualitative comparison of two dyeings which is acceptable to other observers. When he attempts to make this description quantitative by attaching adjectives such as "little", "trace", "much" to the qualitative terms, more difficulties arise, for the pattern which one colourist considers "much duller", a second may consider "very much duller". Many of these differences are attributable to the eye, partly because of the individual colour-sensitivity variation and partly because of variations in the discrimination limen, while others arise from inexperience and the difficulty of obtaining agreement as to the magnitude of differences which are to be expressed verbally. It would be advantageous in settling many matching disputes if colour differences between patterns could be expressed numerically. Methods of measuring colour have already been evolved by the physicist and have greatly advanced in the last few years. The question therefore arises as to whether there is any relation between the colourists' variables and the qualities which are measurable by physical instruments. If such a relation could be found, it might be possible to define the colourists' variables in terms of physical quantities and thus place his nomenclature on an exact basis. The only alternative to this would be to re-educate the dyer to use one of the physicists' systems, the most suitable being the monochromatic system. Actually it is just as easy to define a colour difference in this way as by the present method, and if such a step were taken, personal opinions on patterns could be checked by actual measurements when necessary. Education in the use of this method is greatly assisted by the study of the Munsell colour album, and an attempt has been made to place this system in operation in certain dye-testing laboratories. It seems certain, however, that the dyer will never accept such a system, as he would be unable to obtain from it the information on the comparative dyeing strength of dyes which is essential to him. An attempt must therefore be made to evaluate the dyer's terminology in terms of measurable quantities.

The parallel with the terms used in the monochromatic system of colorimetry indicates that this is the most suitable for interpreting the system employed by the dyer. In the monochromatic system, originally, the pattern was matched by a mixture of white light with monochromatic light, and was specified in terms of three figures, namely: (1) dominant wave-length, which is the wave-length of the spectral light which it is necessary to mix with white light in order to match the pattern; (2) the purity, which is the proportion of monochromatic light in the mixture with white light; and (3) the lightness (brightness, luminosity),

which is the percentage of light reflected by the pattern as compared with a perfectly white surface which would reflect 100 %. This last term is usually referred to by the physicist as relative brightness, but it is not proposed to use this term in the present discussion, owing to the fact that confusion would very probably arise with the colourists' brightness, which is a very different matter. Instead, it is proposed to use the term *lightness*, which has already been suggested by the Optical Society of America (Jones, 1937) for surface colours of the type under consideration.

At the present time, the monochromatic coefficients of a pattern are seldom determined directly, but more usually by conversion from the spectrophotometric or trichromatic data. The monochromatic system shares with the trichromatic system the advantage that a colour is expressed in terms of three figures. Furthermore, it is possible to interpret the monochromatic coefficients mentally in order to obtain an idea of the correct shade. The facility with which this can be done can rapidly be developed by a study of suitable patterns.

§ 2. RELATIONSHIP OF COLOURISTS' AND PHYSICISTS' SYSTEMS

A large number of acid dyes (84) were dyed on woollen piece and were examined by several colourists in daylight and artificial light. The change of shade was expressed as redder, greener, bluer, yellower, qualified by the adjectives trace, little, appreciably, much, very much, in order of increasing magnitude. The final estimates were agreed upon by consultation between the observers, doubtful patterns being re-examined. The patterns were also measured in the Donaldson colorimeter by two observers, average values being obtained. The values in both A and B illuminants were converted into trichromatic coefficients. Points representing the patterns under the two illuminants were plotted on the chromaticity diagram, and the dominant wave-lengths using B illuminant as white diluent determined. From this the change in dominant wave-length when the patterns were viewed in artificial light was derived. The first fact to emerge was that there was complete agreement between the colourists' expression of qualitative change in hue and that revealed by the change in dominant wave-length. This was clear, no matter how small the hue difference, and was especially noticeable in the case of the blues (which change a great deal in purity but very little in hue).

No direct correlation appeared to exist between the magnitude of the numerical difference in dominant wave-length and the colourists' estimate of magnitude of shade change. This is not surprising, as the sensitivity of the eye to differences in wave-length varies over the whole of the spectrum.

These differences in wave-length were therefore converted into the number of perceptible steps of hue difference based upon the data of Wright (1941). Thus the difference between a pattern in daylight and artificial light might be expressed as 10 just perceptible steps. This conversion is not fully justified, for the number of perceptible steps of hue change at a purity of 50 % is probably different from that of the spectrum colours with a purity of 100 %. However, since the essential data for determining the change in least perceptible hue steps for a pattern which changes both in hue and purity are not readily available, the compromise was made. The results of this comparison are given in figure 1.

It will be seen that there is a straight-line relationship between the average number of hue steps in the colourists' groups and the groups themselves when the latter are spaced exponentially. This experiment establishes a close relationship between the shade change as expressed by the colourist and the dominant-hue change as measured by just perceptible steps of difference, and it would therefore appear that the quality of colour which the dyer estimates as *shade* could, without confusion, be termed *hue*, and thus eliminate the former term, which among other colour users has another meaning.

The relationship of the dyers' second attribute, strength, to the physical properties of colour are well illustrated in the following table :—

Table 1. The change of hue, purity and lightness with change of strength

(1) *Naphthalene Scarlet RS*

Dyers' strength, %	Dominant hue (A.)	Excitation purity, %	Lightness, %
0·5	6095	53·6	26
1	6100	64·8	23
2	6100	72·5	19
4	6110	76·0	17
6	6115	76·0	15
8	6100	80·1	13

(2) *Carbolan Green GS*

Dyers' strength, %	Dominant hue, (A.)	Excitation purity, %	Lightness, %
0·5	4916	29	14
1	4904	35·8	9
2	4903	38·8	5
3	4904	36·4	4
4	4907	34·8	3

Considering these results it will be seen that there is only very slight change of dominant hue with an eight-fold change of concentration of dye on the fibre, but that the principal changes are, as would be expected, in the purity and lightness. The purity increases with increasing concentration on the fibre and the lightness diminishes. It will be seen that there is a tendency in the case of Carbolan Green GS for a maximum purity to be obtained, after which there is a reduction in this value. As this last part of the curve is only of importance to the dyer when building up dark colours and blacks, it can be ignored without invalidating the discussion.

The third dimension used by the dyer in assessing the difference between two patterns is that of dullness, and its significance can best be understood by consideration of figure 2, which refers to two dyes of similar dominant hue but different dullness. It will be seen that the locus of the changing lightness and purity with increase of strength of the so-called duller dye lies nearer to

the lightness axis than that of the brighter dye. It is conceivable that dullness might be a difference in lightness only, or a difference in purity only, or a difference in both these functions. Since we have shown above that strength involves a change in both lightness and purity, it is more than likely that this third dimension of the dyer will also involve changes in both dimensions.

As has already been stated, when a dyer makes an estimate of the strength of a pattern, he compares it with a range of dyeings of varying strengths of standard, placing the sample by the side of each individual dyeing of standard until he finds one to which it is equal in strength. Differences which remain are ascribed to dullness or brightness. The question arises as to what physical or psychological qualities are equated by the observer in patterns of equal strength but

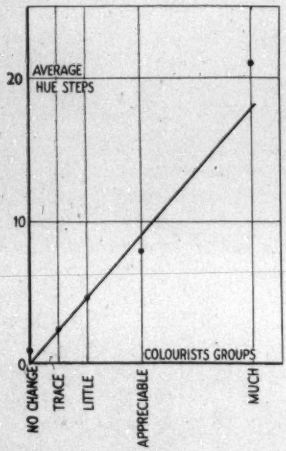

Figure 1.

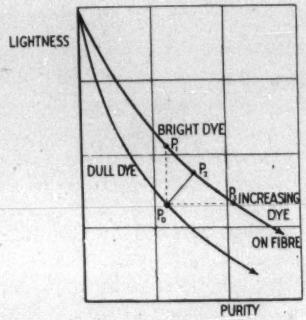

Figure 2.

differing hue or dullness. Three possible explanations can be conceived, namely :—

(*a*) That no physical quality is equated, but that the dyer carries out a mental dyeing trial, which in time becomes subconscious, adding a black dye to the brighter sample as already described. This method can only be employed after much experience, from which the dyer has acquired a knowledge of the manner in which various dyes build up or alter in shade as the strength is increased.

(*b*) That the dyer, subconsciously or consciously, equates patterns with respect to their lightness, choosing patterns of equal lightness as being of equal strength and then describing the purer of the two as the brighter.

(*c*) That the dyer, subconsciously or consciously, chooses patterns of equal purity, and then describes the lighter of the two as the brighter.

The authors of the present paper held divergent views on the matter, so an experiment was devised to investigate the point.

Considering the equation of a dulled pattern with a brighter, P_1, P_2 and P_3 in figure 2 are points representing patterns dyed with increasing quantities of a single dye. If in the pattern P_2 a small quantity of a black dye be substituted for some of the coloured dyestuff, then it can be shown theoretically and practically that both the lightness and the purity of the pattern will be reduced to give the dulled pattern P_D. If a colourist is asked to estimate the strength of P_D against the undulled standard, it is clear that if he is equating purity he will estimate the strength as equal to P_1, which is weaker than P_2, whereas if he equates lightness, P_D will be found equal to P_3, stronger than P_2, and finally, if a mental dyeing trial is carried out, P_D will be found to equal P_2. This forms the basis of the following experiment.

Three acid wool dyes were selected for examination, namely, blue, red and green, and each was dyed in a range of strengths around 1·0 % upon botany serge piece, as described in the experimental section. A neutral grey dye was also prepared, such that a 1·0 % dyeing had a lightness of 10 %. Dulled patterns of the three colours were then prepared and labelled R, S and T, the composition of the dyeings being :—

R 0·98 % Colour + 0·02 % Grey
S 0·96 % Colour + 0·04 % Grey
T 0·94 % Colour + 0·06 % Grey

The dulling effect of the grey was much more pronounced in the case of the red than with the other colours, so that in this case only two dulled patterns, R and S, were examined. The deviation of these patterns in dullness from the standard patterns as expressed in the normal dyer's terminology is approximately:

	Blue	Green	Red
R	trace duller	trace duller	little duller
S	little duller	little duller	much duller
T	appreciably duller	appreciably duller	

The examination of these patterns was conducted in the following manner :—

The ranges of standard were labelled (*a*) to (*g*), (*a*) being the weakest and (*g*) the strongest. The dulled patterns were labelled R, S and T. The observer was confronted with the patterns, usually in a special room illuminated by daylight-type fluorescent tubes specially maintained for colour-matching work. In some cases, where the observer expressed a preference, the examination was carried out in daylight. The observers were told that the unknowns R, S and T were to be compared with the corresponding range of undulled standards, and an estimate given as to which of the standard patterns was equal or nearest in strength. The observer could place the unknown between two of the standards, while the more accurate observers sometimes went further and stated that the unknown lay between two of the standards but inclined more towards one of them. Since the standard range increased in 5 % steps, the placing of a pattern between two standards gave a 2½ % step, and with the more expert observers a 1 % step. Only one observer at a time was allowed to examine the patterns.

Although the present investigation is mainly concerned with the method used by the trained colourist, it was thought that it would be of interest to collect some data from observers who had no experience in colour estimation, and, therefore, the observers who were examined consisted of 26 colourists and 12 non-colourists. The average strengths found by the observers expressed as percentages of the standard dyeing are given in table 2, together with the standard deviation of the means.

It will be noted that the average strength as estimated by the colourists decreases with increasing dullness with all three dyestuffs, but in the case of the non-colourists the estimated strength decreases with increasing dullness with the blue, but increases with the green and red. From these results it appears that the colourist assesses strength upon a different basis from that which is used by the untrained observer.

When these results were compared with the purity and lightness values of the patterns, it was found that the latter were so close together that a reliable estimate of the variation in purity and lightness could not be obtained. The

Table 2

	R		S		T	
	Mean estimated strength	Standard deviation of mean	Mean estimated strength	Standard deviation of mean	Mean estimated strength	Standard deviation of mean
Blue patterns :						
Colourists	97	0·6	93	0·9	90	1·1
Non-colourists	95	1·5	94	1·0	91	1·9
Green patterns :						
Colourists	95	0·8	95	0·8	94	0·8
Non-colourists	98	2·2	101	2·0	104	1·6
Red patterns :						
Colourists	93·5	0·8	92	1·0	—	—
Non-colourists	98	2·2	101	2·6	—	—

values were therefore estimated graphically from values obtained on patterns of a similar type, showing much bigger differences. Details are given in the experimental section, but it may be stated here that the replacement of part of the colour in a pattern by grey causes a reduction in the purity of all three colours.

Regarding lightness, however, it is found that the lightness of 1·0 % dyeings of the red and green is about 15 %, so that the replacement of part of the colour by an equivalent quantity of a grey having a lightness of 10 % in a 1·0 % dyeing brings about a reduction in lightness. With the blue, on the other hand, the lightness of a 1·0 % dyeing is only 7·5 %, so that the replacement of some of the colour by a lighter grey increases the lightness.

Comparing the two sets of results, there appears to be a definite indication that the non-colourist estimates strength on the basis of lightness, a lighter pattern being taken as weaker.

Similarly, it would appear that the colourist estimates strength on the basis of purity, for in all cases the estimated strength is less than the actual content of undulled dye, as is to be expected if this is the method employed.

When the actual values of the strength as deduced from colorimetric measurements on the equal purity or equal lightness criteria are compared with the actual observed strength, however, the agreement is found not to extend to the magnitude of the differences. This is shown in table 3.

From this table it will be seen that although the colourist gives a low estimate of strength, it is not by any means as low as it ought to be if purity is being used as the criterion. Nor in the case of the non-colourist is the strength as high as should be found on the basis of equating lightness.

These facts make it seem very improbable that either of the physical qualities alone is being used for the estimation of strength. The most probable explanation seems to be that the observers rely upon the general impression received from the pattern and do not clearly resolve it entirely into the two physically distinct variables. Obviously, if the patterns do not differ in dullness, it is immaterial whether purity or lightness be used, as either will give the same

Table 3. Strength estimated in various ways

Pattern		By equal purity	Colourist's average estimate	Actual coloured dyestuff content	Non-colourist's average estimate	By equal lightness
Blue	R	91	97	98	95	98
	S	83	93	96	94	96
	T	77	90	94	91	95
Green	R	93	95	98	98	104
	S	87	95	96	101	106
	T	81	94	94	104	109
Red	R	90	93·5	98	98	109
	S	82	92	96	101	120

result, but since changes in lightness are more easily observed, it is probable that this forms the basis of the estimation both by the colourist and non-colourist. When dullness is present, however, the sensations produced by lightness and purity pull in opposite directions. The untrained observer still judges mainly by the more powerful lightness sensation, but his values are somewhat modified by the purity effect. When such an observer undergoes training in colour estimation he comes to realize that his unbiassed judgment over-estimates strength, and therefore learns to give more weight to the purity stimulus or "colourfulness" of the patterns.

It may be thought by the practical dyer that this suggestion attaches too much importance to the physical variables, especially purity. Clearly the observer must equate some quality in the pattern, and this may well be lightness or total light reflected by the pattern, but it can be argued that the further step is solely one of experience. In other words, by practical trial the dyer finds that at first he is liable to over-estimate the strength of dull shades, and therefore learns to apply a correction for dullness in a purely empirical manner.

The third hypothesis which was put forward earlier, namely that the colourist carried out a mental dyeing, adding black to the brighter pattern, is clearly out of the question as far as the non-colourist is concerned, while the colourist gives values which are lower than would be expected on this basis, namely 98, 96 and 94 for R, S and T with all colours. If this method is being used, then presumably the more experienced a colourist becomes, the more nearly correct ought his estimates to become. From the 26 colourists, therefore, eight were selected as having had the greatest experience of this work, and their mean figures, together with those of the remaining 18 colourists, were evaluated. The results are given in table 4, together with the standard deviations of the individual estimates.

From this table it will be seen that the strengths as determined by the expert colourists differ only slightly from those obtained by the less expert, but show no tendency to approach more closely to the true values, but rather the reverse. The small differences found are perhaps to be expected, as all the colourists

Table 4. Comparison of expert colourists, colourists and non-colourists

		Experts (8)		Colourists (18)		Non-colourists (12)	
		Mean	σ	Mean	σ	Mean	σ
Blue	R	97	3·3	97·5	3·1	95	5·1
	S	92	2·6	93·5	5·4	94	3·3
	T	89	3·0	91	6·5	91	6·3
Green	R	95	3·0	95	4·5	98	7·3
	S	94	3·3	94	3·6	101	6·6
	T	93	4·0	94	4·1	104	5·3
Red	R	94	2·9	93	4·3	98	7·4
	S	92	2·7	92	6·7	101	8·8

have had considerable experience in dyeing, so that any change produced by much longer training would be small. The standard deviation figures do indicate that the expert colourists are more consistent than the remaining colourists, and it is further noticeable that whereas the standard deviation of the non-expert colourists increases with dullness, the figures for the experts are almost constant. This indicates that the expert colourists can estimate strength with equal consistency whether dullness be present or not, whereas the consistency of the less expert colourists decreases with increasing dullness. The magnitude of the standard deviations is in good agreement with the view generally held in the dyeing trade, that strength can be estimated visually by a single observer with an accuracy of 5 %, or in the case of highly skilled observers 2·5 %. By the use of several observers accuracy can be increased. Thus, if the strength of the Blue T pattern be taken as an example, the standard deviations of the 26 colourists is 5·4, the strength being 90 (table 2). This means that if any single colourist of this group is asked for his opinion on the strength of this pattern, there are two chances in three that his answer will lie within 85 and 95, a rather wide range with this dull pattern. The standard deviation of the average,

however, is only 1·1. This means that if other groups of 26 colourists were asked to estimate the pattern their average values would lie between 89 and 91 in two cases out of three. The difference between the visual estimate of strength and the content of coloured dyestuff (94) lies outside the limit of experimental error, which makes it appear impossible that a mental dyeing test is carried out.

There remained just a possibility that when used in a mixture with other dyes, a dulled dye containing 96 % of coloured dye by analysis might behave visually as if it contained only 92 %, as the colourist suggests in the case of Red S (table 3). In other words, that the colourist's visual estimate might be more correct in practice than the analytical figure. This was tested by dyeing a mixture shade containing a certain percentage of the undulled red dye with a fixed quantity of a blue dyestuff to give a violet. Further dyeings were then prepared, in which the red was replaced by the dulled red (S) in proportions corresponding to the various strength estimates taken from table 3, namely:—

Equal purity	82 %	Colourist	92 %
Analysis	96 %	Non-colourist	101 %
Equal lightness	120 %		

The resulting dyeings show variations in hue which make the differences more detectable than in the unmixed shades, and when patterns were examined that based on analyses was found to be the closest match to the standard. The colourist's slight under-estimate of the strength of dulled patterns has therefore no practical justification.

Another similar experiment was also conducted, using unknown patterns which were a little different in hue instead of in dullness, as described above. The same Red, Green and Blue standards were used in this experiment, and the hue was altered slightly without altering the lightness and purity of the unknown set of patterns, and these were then submitted to different colourists for their estimate of strength. One immediate, and perhaps surprising, result was that almost without exception the expert colourists refused to give an opinion on patterns which were different in shade by more than a variation judged to be a little different, that is to say, approximately two steps of just perceptible difference in hue. When this proviso had been agreed to, there was a very reasonable agreement among them as to the strength differences. In table 5 are given the results of the examination of three colours (blue, green and red), the first of which was made both redder and greener in three degrees, the second made bluer and yellower in three degrees, and the last made bluer and yellower in three degrees, similarly.

The figures given in table 5 are expressed as percentages, taking 100 as the standard dyeing. It will be seen that little information can be gleaned from these figures, the only points of interest being that in all cases there is a slight tendency for the yellower patterns (greener in the case of blue) to be on the stronger side, whereas the bluer patterns (redder in the case of blue) are weaker. This, together with the fact that the colourists would not assess strength when the hue was appreciably different from the standard, clearly indicates that the hue dimension is distinct from that of strength, whereas strength

Table 5. Effect of change of shade on strength

Patterns increase in degree of divergence from the standard in the order (*a*), (*b*), (*c*)

Standard colour	Hue change	Average estimated strength (*a*)	(*b*)	(*c*)
Blue pattern	redder	98·5	101	100
	greener	101	100	101·5
Green pattern	bluer	99	98·5	97
	yellower	100·5	102·5	107
Red pattern	bluer	100	98·5	—
	yellower	102·5	100	102·5

and the dullness or brightness dimensions are more related. From this work it can be concluded that the dimensions of shade, strength and dullness of the dyers are related to the physical properties of colour in the following manner:—

Shade is probably equivalent to dominant hue in all respects.

Strength and dullness both involve simultaneous changes in purity and lightness, but the colourist when viewing strength uses the physical property of purity as a guide, whereas the non-colourist uses lightness.

§ 3. EXPERIMENTAL

Investigation of the change of shade of patterns in artificial light as compared with daylight

A large number of acid wool dyestuffs were dyed on botany serge piece and the dyeings measured in illuminants A and B in the Donaldson colorimeter by two observers. The average values were obtained in terms of instrument primaries, converted into trichromatic units, and finally, by graphical means, into dominant wave-lengths and purity. The dominant wave-length of the pattern in the two illuminants was determined using illuminant B as the diluent in both cases and the change in hue expressed as number of perceptible hue steps based on the data of Wright (1941). The patterns were also examined by several colourists in a special viewing box so constructed that half the pattern was illuminated by daylight and half by artificial light. The difference in shade was expressed in the dyers' terminology after averaging any variations between the observers.

Estimation of the strength of dull patterns

In order to carry out this series of tests, a range of patterns was prepared on botany serge dyed with three dyestuffs—Lissamine Fast Red BG, Lissamine Green 2GS, Solway Blue SES. These three dyestuffs were dyed at strengths of 0·85, 0·90, 0·95, 1·00, 1·05, 1·10 and 1·15 %, calculated on the weight of the material. The technique employed was carefully controlled as to variation in weight of material, dyestuff, liquor strength, temperature and time. These series were inspected and found to run in the correct order, and apart from one or two minor criticisms by the more expert colourists, could be taken as representative of such a range of strengths.

Three patterns were then prepared in the case of the blue and green, and two in the case of the red, which diverged from the standard colours in dullness by having a small percentage of neutral grey dyestuff dyed with them. The neutral grey dyestuff was prepared by matching the shade of the grey patterns given in the Munsell colour chart, and consisted of a mixture of 20·25 % Lissamine Fast Red BG, 18·09 % Lissamine Fast Yellow 2GS, 21·68 % Solway Ultra Blue BS, and 40·00 % diluent. For the purpose of this investigation the mixture was always made up in solution form and the diluent was therefore water. The dulled patterns were prepared as follows:—

Blue	Green	Red
%	%	%
R 0·98 Solway Blue SES	R 0·98 Lissamine Green 2GS	R 0·98 Lissamine Fast Red BG
0·02 Neutral Grey	0·02 Neutral Grey	0·02 Neutral Grey
S 0·96 Solway Blue SES	S 0·96 Lissamine Green 2GS	S 0·96 Lissamine Fast Red BG
0·04 Neutral Grey	0·04 Neutral Grey	0·04 Neutral Grey
T 0·94 Solway Blue SES	T 0·94 Lissamine Green 2GS	
0·06 Neutral Grey	0·06 Neutral Grey	

These patterns were examined by the 38 observers in the manner already described. The results are given in tables 2 and 4.

Determination of purity and lightness of the patterns

The actual patterns examined by the colourists were first examined in a Donaldson colorimeter, but the trichromatic values of the R, S and T patterns lay so close together that they could not be used as a satisfactory basis for the determination of the lightness and purity of the patterns. A fresh series of patterns of each colour was then prepared, consisting of dyeings of 0·5, 0·75, 1·0, 1·25 and 1·50 % of the colour, and three mixtures of the colour with grey, namely 0·90 % colour plus 0·10 % grey, 0·80 % colour plus 0·20 % grey, and 0·70 % colour plus 0·30 % grey. These patterns were measured on the colorimeter by two observers and the mean values obtained. These are given in the tables below;—

Table 6. Lightness and purity of patterns dyed with Lissamine Fast Red BG

Patterns	Trichromatic values x	y	Lightness	Excitation purity
0·5 % Red	460	278	18·6	43·5
0·75 % „	477	279	16·5	46·2
1·00 % „	496	281	14·8	49·0
1·25 % „	506	280	13·4	51·2
1·50 % „	517	281	12·3	53·0
0·90 % „ +0·10 % Grey	457	275	12·2	44·0
0·80 % „ +0·20 % „	439	275	10·6	40·7
0·70 % „ +0·30 % „	420	275	9·7	37·0

Table 7. Lightness and purity of patterns dyed with Solway Blue SES

Patterns	Trichromatic values x	y	Lightness	Excitation purity
0·5 % Blue	207	221	12·7	48·2
0·75 % ,,	194	196	9·4	56·0
1·00 % ,,	189	184	7·5	59·5
1·25 % ,,	185	178	6·6	61·5
1·50 % ,,	183	174	5·7	63·0
0·90 % ,, +0·10 % Grey	199	202	8·0	54·0
0·80 % ,, +0·20 % ,,	204	213	8·2	50·5
0·70 % ,, +0·30 % ,,	207	226	8·4	47·5

Table 8. Lightness and purity of patterns dyed with Lissamine Green 2GS

Patterns	Trichromatic values x	y	Lightness	Excitation purity
0·5 % Green	220	322	21·2	33·0
0·75 % ,,	211	313	17·6	37·0
1·00 % ,,	205	308	14·7	39·8
1·25 % ,,	201	306	12·8	41·2
1·50 % ,,	198	302	10·7	43·0
0·90 % ,, +0·10 % Grey	206	349	13·6	36·5
0·80 % ,, +0·20 % ,,	216	348	12·4	33·0
0·70 % ,, +0·30 % ,,	225	347	11·5	29·6

From the data given in the above tables the purity and lightness of patterns containing smaller quantities of grey were determined graphically.

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A SENSITIVE METHOD OF ADJUSTING LENSES TO CAMERAS

By E. W. H. SELWYN

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ABSTRACT. The adjustment for focus of lenses to cameras is rendered inexact, partly by the depth of focus of the eye, but mainly by its facility in accommodation. This applies even when ground glass is used as a screen, for under the usual conditions of use the directly transmitted light is much more important than overall ratio of directly transmitted to diffused light would suggest. It has been found that a diffusing screen made by grinding down thin flashed opal glass does not have this disadvantage, and that when it is used with a sloping test-object, so that the aerial image is at an angle to the screen, the adjustment of focus may be made by adjusting the image to be symmetrical about suitable reference marks. The precision of this setting is very high.

§1. INTRODUCTION

At the Optical Convention of 1926, Mr. T. Smith said, "I have often been surprised at the manifold use made of any tolerance in photographic work—for example, latitude in the position of focus. The lens computer, let us suppose, uses it all for chromatic aberration, and then, as though it were all still available, uses it again for spherical aberration, and still again for curvature. The camera maker takes full advantage of it to get latitude in the fitting of focusing scales, also to condone variations of position of the plate surface in different slides or of the focusing screen, and again to depreciate the importance of want of rigidity in the camera front. It is also supposed to cover any errors in fitting the lens into the camera. Lastly, the photographer himself considers it at his service should his focusing be inexact, and further relies upon it to give depth of focus. Unsharp images on the plate may arise from all these causes simultaneously, and there is no reason to suppose that because the sources are numerous there will be a distinct tendency for one fault to neutralize another. I can only express surprise that the work of photographers is as successful as it appears to be."

The picture is not now quite so black as it was then painted, but the possible errors still make it worth while to reduce any or all of them. Part of this must be accomplished by the lens designers and part by the engineers who make the cameras. This leaves the adjustment of the lens to the camera as a problem to be tackled by the physicist.

The object when setting a lens on a camera is to adjust it so that photographs taken by it shall be as sharp as the lens can possibly give. The only way of doing this with absolute certainty is to choose some suitable test object and to make photographs of it for a series of different settings of the lens. The setting which gives the best photographs can then be chosen. Such a laborious process is at the least undesirable as a manufacturing operation, and lenses are almost

invariably focused visually. Visual focusing is usually carried out for a very small region only, in the centre of the field, whereas the photograph itself covers a relatively large field. Owing to the curvature of field, it may be necessary to move somewhat away from the position of best focus at the centre of the field, in order to secure adequate definition elsewhere. This introduces a difference additional to any colour difference.

The setting of the lens giving the best results depends also upon the nature of the test object. Most lenses will give two types of axial image of a point source, namely, a very sharp image with a flare of light round it and, at a short distance from this position, a rather bigger image without flare. Consequently, if the object to be photographed is one of contrasty fine detail, the first setting is preferable, but for coarser detail, with less contrast, the second setting is desirable.

The solution to the above difficulties is relatively simple. A number of cameras should be fitted with lenses by the photographic method of triai and error, and then be tested by the very practical method of taking ordinary photographs with them. When these cameras are finally adjudged satisfactory, they should be used for adjusting the visual focusing instrument, so that whatever the nature of the instrument and its test object, there is a reasonable expectation that a camera focused on it visually will give good results photographically. A number of cameras must be used for this purpose. Different adjustments may be necessary for different cameras, owing to inevitable small variations in manufacture of both cameras and lenses, and the focusing apparatus should be set to an average amongst these different adjustments. The best possible allowance is thus made for small variations.

§ 2. COINCIDENCE ALONG AXIS BETWEEN AERIAL IMAGE AND CROSS-WIRES

If cross-wires or other locating device are fixed at the focal plane of a camera, an image of a distant test object can be made to coincide with it not only laterally, but also in the direction of the axis. In this arrangement some magnification of the image in the focal plane is necessary, and may be obtained with a simple or compound microscope, whichever is the better. It may be necessary to use a collimator if a test object at a long distance is inconvenient, in which case a collimator of long focal length should be used to keep down the effect of its aberrations.

An alternative arrangement uses a test object in the plane of the film of the camera. The image of this, seen through the camera lens, is examined by means of a telescope.

If a is the size of the image or test object at the focal plane of the camera, and b the size of the virtual image of it, at 250 mm. from the eye, then by general optical theory as described, for instance, by Conrady (1929), or as may be worked out easily enough for any particular case,

$$b/a = 250/d(F.No),$$

where d is the diameter of the pupil of the eye in millimetres and $F.No$ is the focal ratio of the camera lens. The diameter of the pupil varies very considerably according to the brightness of the light, and there is some variation

from one observer to another. In ordinary optical instruments, a 2-mm. iris is frequently regarded as a standard. Certainly, the best performance of the eye in respect of sharpness of vision occurs at an aperture of about this value. For a 2-mm. pupil, the appropriate magnifications for various aperture ratios are given in table 1.

Table 1. Minimum magnification for viewing aerial image

Aperture of photographic lens	F/1·9	F/3·5	F/4·5	F/6·9	F/8	F/14
Magnification	66	36	28	20	16	9

It will be obvious that such magnifications can only be obtained easily with compound microscopes.

Experiments indicate that the depth of focus of the eye is, for iris apertures of 3 mm. or so, very roughly about the same, or rather larger, than that calculated on the quarter-wave basis for a perfect lens of the same aperture and focal length. These experiments also showed that it was very difficult to fix the accommodation, and they were extremely tiring to the eye. There was all the time a strong tendency for the eye to jump from the reference marks, upon which the eye was accommodated, to the subject. This subject was moved away from a position close to the reference marks until it was just perceptibly unsharp. But unless

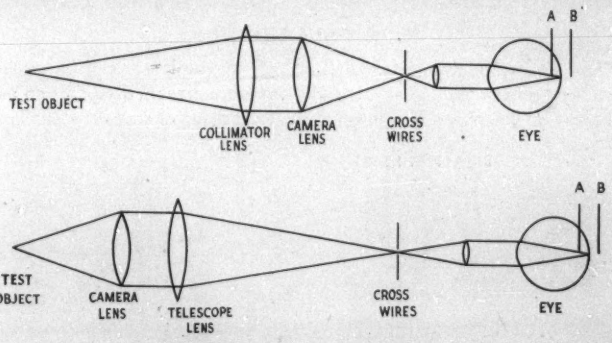

Figure 1. Focusing arrangements using aerial image.
If AB is the depth of focus of the eye, the cross-wire image may be at A and the test-object image at B, or vice versa, without noticeable difference between them.

attention was maintained rigidly upon the reference marks, and the detail of the subject immediately surrounding them, the eye automatically refocused upon whichever was looked at. Some of the difficulty appears to be due to the very narrow angle of sharp vision.

Suppose that in the arrangements of figure 1 the position of focus, in the eye, for the image of the test object is at A and the image of the cross-wires or other locating device is at B, then if the distance between A and B is the theoretical depth of focus, the two images are indistinguishable from perfectly sharp ones. Obviously the positions of A and B can be interchanged without affecting the

sharpness of the images. This means, in effect, that the position of the camera-lens can be moved over a distance equal to twice the depth of focus without any change being observable.

Table 2. Minimum range with fixed accommodation of the eye

F/1·9	F/3·5	F/4·5	F/6·3	F/8	F/14
0·02	0·05	0·09	0·18	0·28	0·88 mm.

As has already been pointed out, fixation of the accommodation to the degree required for the above accuracy is difficult and extremely tiring to the eye. The variation of accommodation possible is very great, but moderate shifts of the distance of accommodation are recognizable, even with monocular vision, and in the ordinary way for a skilful operator the range in focusing with the above disposition of apparatus would appear to be three or four times the above values.

It can happen that a particular configuration of the image at the focus can be recognized with greater precision than the depth of focus would suggest. Thus, for a particular lens, the precision of setting upon some particular point near the focus may be very high. Unfortunately, this easily recognizable point is not likely to be at the same distance from the photographic focus for all lenses of the same formula, owing to inevitable small variations in manufacture.

§ 3. PARALLAX METHOD

The well-known parallax method of locating the image can be modified to form a very convenient and at first sight very attractive method. If an arrangement is set up whereby a test object at the focal plane of a camera lens is viewed

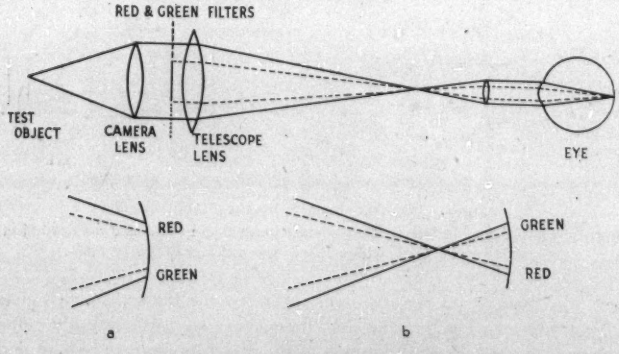

Figure 2. Modified parallax method.

If the image is out of focus in one direction, the red image appears to the right of the green (*a*), and when out of focus in the other direction the red image is to the left of the green (*b*).

through the camera lens by means of a telescope, the objective of which is covered over one half by a green filter and over the other half by a red filter, the image will be seen doubled except when in focus. The doubling is made

obvious by the difference in colour, and is particularly emphasized if the middle part of the telescope objective is stopped out. When the focusing is exact, the double colouring disappears and the image becomes yellow.

As the object of the process is to set two images to coincidence, one might expect very high accuracy. For coincidence settings, the repetition error is of the order of ± 5 seconds of arc, from which it may easily be calculated that the average focusing error should be $F.(F.No)/100{,}000$, where F is the focal length of the lens. However, although the repetition accuracy is high, systematic variations from one observer to another differ by as much as 25 seconds. Moreover, the chromatic aberration of the eye can lead to serious errors. Guild (1917) estimates that parallax errors of 25 minutes can occur when red and blue light are involved. For red and green the error is likely to be about 5 minutes. This chromatic error is peculiar to this method, but it also suffers from the same difficulty of maintaining fixed accommodation.

§4. GROUND GLASS AS FOCUSING SCREEN

Ground glass is commonly used as a focusing screen. It is usually claimed that it "fixes" the image more closely than is possible with cross-wires or a plain glass marked with scratches. The argument is that, owing to the scattering of light by the ground surface, it is not possible to see the direct image given by the camera lens through the glass. Thus the variation of accommodation in moving the eye from the cross-wires to the image is eliminated because the image is necessarily coincident with the ground surface. However, this may not be strictly true, as may be seen from the following considerations.

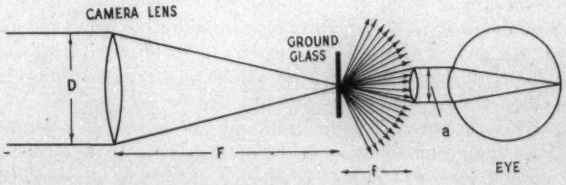

Figure 3. Ground-glass focusing screen.

The ground glass diffuses light in all directions, so that only a fraction of it enters the eye, but the undiffused light, which maintains its original direction on passing through the ground glass, all enters the eye if the magnification is chosen correctly.

The light transmitted by ground glass consists of two parts, one of which is transmitted directly without any change except for a reduction in intensity, and a second part which is diffused by transmission through the ground glass. This may easily be seen by looking at a distant lamp filament through a piece of finely ground glass held at arm's length from the eye. The glass itself will appear bright as a result of the diffusion of the light caused by the scattering properties of the ground glass, but also the lamp filament will be seen quite sharply, but of diminished intensity. Figure 3 represents the case where the image thrown by a lens on to ground glass is viewed by a magnifying lens of

relatively low power, and reference to it will make it clear that the directly transmitted light is much more effective than the diffused light, because the diffused light is spread over a complete hemisphere, at least, and only a small fraction of it can enter the eye.

The relative importance of the undiffused and diffused light may be estimated in the following way:—

Suppose that a fraction x of the light falling on the ground glass is transmitted without diffusion and that $(1-x)$ is diffused, and that the diffusion is complete. Let the total flux into the image on the ground glass be B, so that the total amount of diffused light is $(1-x)B$. The eye will only pick up a part of this. If the diameter of the pupil of the eye is d and the focal length of the magnifying lens is f, the amount of diffused light picked up by the eye will be $(1-x)Bd^2/4f^2$. The directly transmitted light will amount to xB, but this is spread over a solid angle $\pi D^2/4f^2$, and the eye can take in light over an angle of only $\pi d^2/4f^2$. Consequently the amount of undiffused light picked up by the eye is xBF^2d^2/f^2D^2, and the ratio of undiffused to diffused light is therefore $4xF^2/(1-x)D^2$, which is $4x(F.No)^2/(1-x)$, since F/D is the focal ratio of the camera lens. With an F/4·5 lens, therefore, the ratio of undiffused to diffused light is just about $80x/(1-x)$, so that if the ground glass transmits only 1 % undiffused light, the amount of undiffused light actually received is almost equal to the diffused. The image produced by undiffused light is very much sharper than that produced by the diffused light, and presumably, in effect, brighter, but this will be offset, so far as the above calculations are concerned, by the fact that the ground glass is not a perfect diffuser, but such that more diffused light enters the eye than the above calculations suggest. The estimate given, however, is probably of the right order of magnitude. Some important conclusions follow from this. Since, for a considerable fraction of the light in the image the ground glass acts as if it were polished, the image is not located any more precisely than with cross-wires, except in so far as the pitted glass surface may be more effective than cross-wires in fixing the accommodation of the eye. Moreover, it is customary to use magnifications of only seven to ten times when focusing on ground glass, owing to the obtrusiveness of the pitted surface at higher magnifications. Unless the amount of directly transmitted light is considerably less than 1% it is necessary to use the magnifications given in table 1, otherwise the full aperture of the lens will not be used for the most important part of the image. The effective aperture may be as low as F/9, with the ordinary type of magnifier, and the possible range of focusing, with absolutely fixed accommodation, 0·3 mm.

§5. NEW METHOD OF FOCUSING

It has been shown that all methods of focusing lenses on cameras suffer from a limited accuracy due to the depth of focus of the eye, and that this accuracy can be reached only by the most rigid concentration, to avoid changing the state of accommodation of the eye.

It is clear that the idea behind the use of ground glass is a good one, since if ground glass allowed no light through except diffused light, the difficulty of the eye moving from the reference marks to the lens image would not arise.

After several trials of different materials it was found that a very good screen could be made of thin flashed opal glass, the opal layer of which was ground down with flour of emery until the filament of a projector lamp at about 15 feet distance could just be seen through the opal glass when held at arm's length. The *grain* of this screen is less than that of ground glass of equivalent diffusing power. In fact its properties are not very different from those of a fast photographic film, in the sense that the image seen on it is closely similar to that obtained, after development, if an exposure is made on film to the same test object, with the film in the same position as the screen.

The advantage of such a state of affairs will be fairly obvious. There is evidence that the position of best focus for photographs is not necessarily the same as for best visual focus, and it is also very probable that a lens which gives a satisfactory visual image may not give a good photograph and vice versa. Consequently, a screen which approximates to photographic materials in its properties should lead to better photographic focusing. It can also be used, with advantages in some directions, for testing lenses, but in this case difficulties

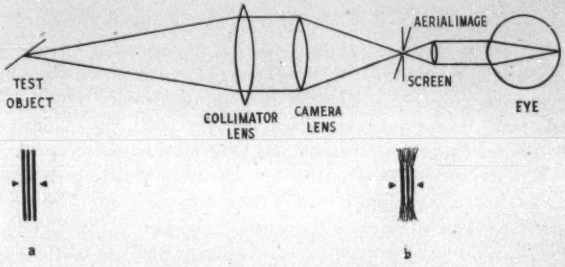

Figure 4. New method of focusing.
The image *b* of the test object *a* is sharp only over a limited region. This is arranged to lie between the arrow-heads. The lens is then in focus.

arise if it is made to replace film directly owing to the differences in scattering at different angles to the normal.

Naturally, owing to the uneven surface of the glass on which the opal layer is coated, only small pieces can be made.

If such a screen were used in a normal arrangement, the usual procedure would be to move the camera lens rapidly through the region where the image was fairly sharp, and set the lens at a position guessed to be that giving the sharpest focus. By inclining a suitable test object at an angle to the optical axis, this change from out-of-focus to sharp and back again can be converted from one in time to one in space. This arrangement, when used with a photographic plate to receive the image, seems to be well known. It appeared in a book by Eder dated 1911, and an almost identical arrangement was suggested by Claudet in 1851. The reverse arrangement in which the test object is at right angles to the optical axis and the photographic plate inclined has been used by Wetthauer (1921) and Conrady (1926). The apparatus thus consists of a collimator, with the test object inclined to the axis. Following the collimator

is the camera lens, which forms an image of the test object near its focal point, which, since the test object is inclined, is also inclined to the axis. This image is received upon the above-mentioned diffusing screen at right angles to the axis. Thus the image is diffuse at the top, sharp in the middle, and diffuse again at the bottom. If the lens is moved backwards or forwards, the position of the sharpest part of the image will move up or down, and when the lens is set to its best position, the sharpest part of the image will be at the centre. The centre is marked on the test object at the position where it crosses the axis. This is done because lenses may throw the image of an axial point off the axis. This effect is not serious on a camera, but would obviously throw out the setting of a lens on this type of apparatus if the sharpest part of the image were made to fall on the centre of the screen, and not the centre of the image of the object.

As a result of numerous experiments, it has been found that the best test object consists of three vertical parallel lines with a width equal to the transparent spaces between them. The image on the diffusing screen is observed with a low-power microscope or magnifying lens, which does not need to be of higher power than about 10×.

As the aim is to find the sharpest part of the image, the problem of accommodation does not arise acutely. If the eye is not accommodated precisely to the setting of the microscope, the whole image appears somewhat less sharp, but the sharpest part of the image does not shift appreciably for different states of accommodation. In fact, the eye accommodates itself automatically to the setting of the microscope, for there is only one image to focus upon. Except in cases where the correction of the camera lens is such as to give a noticeably unsymmetrical image, the setting is made by adjusting the image to be symmetrical about the arrow-heads which mark the centre of the test object. The precision is remarkable. After a few minutes' instruction in the use of the apparatus, operators who had not before used optical instruments were able to set F/4·5 lenses with a repetition accuracy of ±0·03 mm.

Different observers do vary in the setting they choose, although the differences between observers are very small. This appears to be due to differences in acuity. If the acuity is high the out-of-focus "tails" are longer, so that when the falling-off in definition is unsymmetrical, as it always is to some extent, owing to the presence of spherical aberration, then the position of the estimated centre of symmetry will vary for different observers. The quality of the image given by the lens influences the accuracy with which the position of best focus can be found only to a moderate degree, unless the quality is very bad.

In practice the inclination of the test object to the axis should be low. The lowest which can be used with any satisfaction is about 10°, for at lower inclinations the loss of light by reflection becomes serious. Each focusing instrument should be constructed to suit the particular lens with which it is to be used. But as a general rule it may be taken that when the inclination of the test object is 10°, the focal length of the collimator lens should be about five times that of the camera lens, and the magnification of the image on the screen about seven times. For the larger aperture lenses the collimator may be lengthened, and the magnification increased. There is, however, some considerable latitude in dimensions.

It is frequently convenient for the diffusing screen to be capable of moving

backwards and forwards along the optical axis, but in such cases it has been found very desirable to arrange that this is done without rotation of the screen. If the screen moves laterally with respect to the image, the motion of the grain of the screen attracts attention away from the image.

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DISCUSSION

Mr. C. F. SMITH. I have used a device for the measurement of focal length which was similar to that described by Mr. Selwyn, with the exception that the screen was opaque and inclined at an angle of about 15° to the optical axis. A finely divided scale was placed at the principal focus of the lens under test, followed by a lens of comparatively long focal length, the screen being placed at the principal focus of this lens. Owing to the great inclination of the screen, the projected scale was very much lengthened, the lines being blurred except at one point, the position of which was dependent on the focal length of the test lens. At one time I was concerned in the production of plastic lenses, for which it was necessary to devise means of testing sufficiently fast to cope with the high scale of production. The above system proved very suitable for meniscus camera lenses, and the apparatus was set up by means of a master lens, which had previously been tested and approved for focal length, curvature of field, astigmatism, etc. Differences of focal length of subsequent lenses were indicated by a movement of the sharply defined portion of the scale, and if this moved beyond previously drawn marks representing the tolerances on focal length, the lens was rejected.

One advantage of the system is that there is a plane of good definition of some extent, and the result is not seriously affected by bad edging of the lens or by faulty centering in its mount. The system is similar to that employed some years ago in some apparatus made by Hilger's for adjusting the position of objectives in box cameras, or for fixing the position of the focusing scale on folding cameras. The transmission-screen device used by Mr. Selwyn possesses obvious merits, enabling the maximum accuracy to be obtained. I was interested in Mr. Selwyn's reference to the rotating-vane method of registering the position of best focus, since I used this device during the Great War for testing the focal length of binocular lens components : I believe it is at least a century old.

The Rev. REES WRIGHT. I suggest that the Foucault test might with advantage be employed for testing the focus ; this method is almost universally employed for focusing astrographic telescopes. I believe that the " knife-edge " can be reduced to a clean-cut slip of tinfoil cemented to an exposed and clean plate placed in the plate-holder. It seems to me, too, that the use of an independent collimator introduces possible errors, though probably the autocollimator method would not be applicable to lenses as small as those in cameras.

Mr. E. F. FINCHAM. Mr. Selwyn has referred to the possibility of examining the image upon the actual film in the camera. This method cannot be applied in the case of those cameras, such as the sub-standard cinematographs and miniatures, where the back cannot be removed. One wonders whether, in these cases, where no focusing-screen method can be used, one of the methods employed for the objective measurement of refraction of the eye would not be of some value. Retinoscopy would detect quite small errors in focus for distances within and up to about 1 metre, and by the use of an

auxiliary lens for objects at infinity. Another method, if the instrument is available, is an objective optometer. I have used the Coincidence Optometer, which has been described in these *Proceedings*, for this purpose. This instrument will detect an error of less than 1/10 dioptre, i.e. in the case of a camera with lens of 2 inch focal length, an error in the position of the screen of the order of 0·01 inch for an object at infinity.

AUTHOR'S reply. I have hitherto believed that the arrangement described by Mr. Smith was not accurate when the lenses tested upon it were badly edged or centred, but on further consideration I agree that it is, in fact, satisfactory. One of the greatest difficulties with single lenses is that their aperture is low. This means that the amount of light available is small and the depth of focus considerable. Thus, whether one uses an opaque screen, as in Mr. Smith's arrangement, or a translucent one, as in mine, the accuracy possible is limited by the depth of focus and the ease of use by the low illumination. On the whole, the translucent screen probably gives the better results. I am glad to know that Mr. Smith is in favour of the use of master lenses for setting the testing apparatus.

Unfortunately, camera lenses are not so well corrected as astronomical telescopes, and the Foucault test cannot be very well used for focusing, as suggested by Mr. Wright. It can be used for testing for aberrations in the way described by Kingslake (1937),* but that is a different matter. Its disadvantages for focusing camera lenses are that it is inherently too sensitive and, for that reason, requires a delicacy in operation which is not to be expected in manufacturing processes. The use of an independent collimator produces no serious errors provided that its focal length is several times that of the camera lens.

The methods which Mr. Fincham suggests would clearly be of value when the focal plane of the camera is inaccessible, but in general the focal plane is arranged to be accessible at some stage during manufacture. Thus, although the idea of looking at the image through the camera lens would appeal strongly to the owner of a camera who did not wish to disturb the adjustments of it, it is doubtful whether manufacturers would consider it of real value. Manufacturers have also become accustomed to greater accuracy of setting than 0·01 inch on 2-inch lenses.

* *Proc. Phys. Soc.* 1937, **49**, 376.

CONDITIONS FAVOURING THE START OF AN ARC DISCHARGE BETWEEN COLD ACTIVATED ELECTRODES AT 50 CYCLES PER SECOND

BY M. PIRANI, PH.D., F.INST.P.

Communicated by Dr. C. C. Paterson, F.R.S., 17 *June* 1942

ABSTRACT. It has been shown by experimental investigation that the start of an alternating current (50 cycles per second) arc-discharge between two activated electrodes in a rare-gas filling takes place in two stages, the first being a glow discharge with high voltage and low current, the second a change-over to the arc-discharge proper with low voltage and high current. The change-over is favoured by electrodes comprising a pellet of special physical structure and chemical composition producing small amounts of "active barium". This has been proved by experiments with discharges under conditions where no positive column is present.

In long tubes where the main part of the voltage is taken up by a positive column, the start of the glow discharge is hampered by wall charges. These can be removed by auxiliary devices applied on the outside wall of the tube and preferably extending over its whole length. A design dispensing with such auxiliary devices has been described, by which the wall discharges are removed through almost continuous glow discharge between two auxiliary electrode wires extending through the greater part of the length, one or both being connected through small condensers to the main electrodes.

§ 1. INTRODUCTION

ONE of the major problems in the design of the modern mains-operated low-pressure arc discharge lamps of the positive-column neon, sodium or mercury fluorescent type, is that of starting, especially if the electrodes are not separately heated (von Engel and Steenbeck, 1932 and 1934 ; Gabor, 1939).

By a systematic experimental study, made between the years 1936 to 1940, of the starting phenomena in different kinds of discharge tubes, with and without activated electrodes, the conclusion has been reached that there are two distinct problems which must be treated independently. The one, related to surface chemistry, concerns the electrodes alone; the other, which concerns the walls of the tube alone, is related to surface physics. This conclusion has been reached indirectly by the conflict of experimental evidence.

The present paper gives the results of this research without mentioning the actual difficulties met with during the investigation itself.

It is proposed to discuss the problem in two parts: firstly, the start of an arc discharge between electrodes separated by a distance small compared with their dimensions (Alterthum, Reger and Seeliger, 1920), and secondly, the starting of an arc discharge between electrodes located at the ends of a tube, long as compared with its diameter (Pirani, 1931).

§ 2. ELECTRODES CLOSE TOGETHER

Figure 1 shows a model discharge bulb for the first part of the investigation. Two tungsten coils (figure 2) made of 0·15 mm. wire and having 25 turns (of which only 5 are shown in figure 2) on a 1·5 mm. mandril surround rod-shaped cores made of a highly active barium compound. The coils are located 10 mm. apart in a glass bulb of 60 mm. diameter.

Obviously in this model the positive column is negligible, and only the gas-

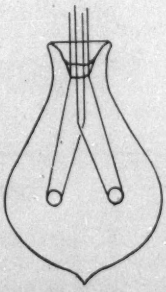

Figure 1.

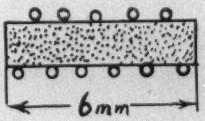

Figure 2.

filling and the material of the electrodes can affect the starting conditions. When this bulb is filled with a neon-helium mixture containing 25% helium, and a 50-cycle voltage is connected to the electrodes through a 2500-ohm resistance, a glow discharge suddenly appears when the voltage reaches 235 to 240 volts, the current being about 20 mA. A violent spluttering of the small electrodes is then observed.

When, however, the experiment is repeated with a lower series resistance, say 750 ohms, the electrodes show localized hot spots a few seconds after the glow discharge has started, and the voltage drops to about 50 volts; the resistance limits the current to about 250 mA. It may be stated here, subject to further discussion, that the limiting resistance at which the change-over from glow to arc discharge takes place is higher, or, in other words, the limiting current is lower, if a compound of higher "activity" is used as a core for the electrode coils. This is a representative test for what will be called *activity* in this paper.

When, instead of the neon-helium mixture, argon at 9 mm. pressure is used and a drop of mercury giving a partial pressure of the order of a few hundredths of a millimetre is placed in the bulb, retaining the same electrodes and series resistance, the glow discharge starts at 110 volts and the change-over to an arc discharge takes place and the voltage falls to about 20 volts, the current being 0·15 amp. The starting voltage of the glow discharge is governed by the ionization potential of the gas and by the cathode and anode drop.

When barium-activated electrodes are used, the particular compound seems to be unimportant. This is explained by two facts : (*a*) the necessary electrons for the self-supporting discharge are liberated by ion impact on the momentary cathode, (*b*) the kinetic energy of the ions is in any case large enough to destroy every compound and liberate the necessary barium to form a monolayer on the tungsten electrode (see Benjamin, Cosgrove and Warren, 1937).

The second phase, including the change-over from glow-discharge to arc, does not seem very easy to explain (von Engel and Steenbeck, 1932 and 1934; Ramsay, 1939), though the present investigation shows a way to control it. A qualitative description of the requirements for such an electrode will now be put forward; it has served the author as a working hypothesis which, so far, has proved to be consistent with the experimental results. In order to make the start of the glow discharge possible at the low voltage with alternating current, the cathode drop must practically disappear. The lowest possible cathode drop in all gases is with a barium-activated cathode. In order to prevent the stock of barium from evaporating through the effect of ion impact, thus leaving the cathode inactive, a continuous formation of monolayers has to take place. This means slow but steady barium production, and, therefore, a compound of barium must be used which retains the barium with a certain force, which has sufficient stock of barium, and whose physical structure is such that the barium can migrate towards the surface of the electrode wires.

In order to favour the change-over from glow to arc with a cathode initially cold, an inhomogeneous structure of the electrode is further necessary, and the surface of the electrode should have slightly different electron emission at different points, so that by a self-multiplying process the hot spot with 1 v. cathode drop is formed. The barium "storage tank" must, therefore, be able to withstand high local temperatures without destruction or evaporation, and the amount of barium released must not unduly increase, so as to empty this storage tank by loss of all its barium. One solution which was found to meet the above requirements was :—

(*a*) To use a tungsten-wire spiral closely surrounding a rod extruded from a mechanical mixture of a stable barium compound with a substance having a high

affinity for oxygen and other gases which would "poison" the activity of the cathode when they are liberated during the running of the tube.

(*b*) To bring this compound to reaction after the discharge vessel has been evacuated and filled with clean rare gas. By the reaction a certain small amount of what is commonly called *active barium* (and is probably a mixture of barium metal and low oxides) seems to be formed, having a tendency to form alternating monolayers with the low oxide on top. This migrates over the tungsten wires and activates them. The necessary inhomogeneity is given by the porous and granular structure of the pellet.* The slight electrical conductivity (quickly increasing with temperature) which the pellet shows after the reaction provides a variable contact with the spiral, and this enhances the formation of a hot spot, because of the slight variations of work function at different places. The hot spot also wanders about on the spiral from time to time during the running, because vibrations may affect the point of contact of the pellet with the spiral.

Suitable barium-oxide compounds which are stable and not sensitive to moisture are $BaCrO_4$ and $BaOAl_2O_3$. The metals used are Zr, Si, Ti, Ta and highly reactive alloys like FeTi, all of them forming irreversible compounds with oxygen and some of them with nitrogen and even hydrogen.

Aluminium or magnesium powder could also be used as a reducing metal, but then a very violent evaporation of the metal takes place and the reaction produces too large an amount of barium, which in turn evaporates, leading to the quick exhaustion of the cathode.

Similar results were also obtained by the use of a tantalum spiral, in which case the admixture of a reducing agent to the pellet becomes unnecessary, because the metal of the spiral acts as a reducing agent; but more consistent results were obtained with the powder pellet containing the reducing metal.

Instead of barium, compounds of strontium and calcium were tried, when the running voltages were higher, corresponding to the higher work function of strontium and calcium as compared with barium.

Though the change-over is just as reliable in this case, the higher energy loss on the electrodes causes the useful life of such electrodes to be shorter. The average order of loss on the electrodes, when the hot spot having a temperature of about 1500° C. is formed, is 3·5 to 4 watts per electrode with a current of 400 mA.

This is very small compared with the energy which would be needed to create the necessary vacuum electron emission of an indirectly heated cathode to carry the same current. This would be of the order of 15 to 20 watts per electrode and, of course, would not be self-supporting, but would have to be sustained by a separate source of energy, because the slightest inhomogeneity, resulting in an uneven heating of the cathode and, of course, the formation of a hot spot, would immediately destroy such an electrode.†

The voltage-time oscillograms of the change-over with a tube starting from cold, and the tube with heated electrodes giving full vacuum emission, can be easily distinguished because the first has a starting kick in every half-period, resulting from the fact that during the transition time from positive to negative

* In order to make the pellets by extrusion, the powder particles must be smaller than 0·1 mm. and a binder should be used which evaporates without leaving appreciable amounts of carbon behind, e.g., 50 % monostyrene solution or 25 % solution of ethyl cellulose in pine oil.

† The same relation holds for a current of 250 mA.

the hot spot cools down a little and henceforth the electron emission drops (figure 3). This results in an increased running voltage until the original equilibrium conditions with the higher emission are re-established.

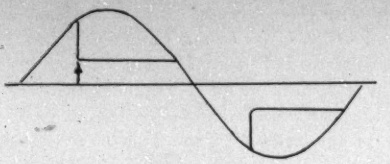

Figure 3.

The other tube with indirectly heated electrodes giving full vacuum emission would have no starting kick in the curve.

§ 3. ELECTRODES AT CONSIDERABLE DISTANCE

Having now discussed the starting conditions of the arc between unheated electrodes at a small distance from each other, we can consider the problem of starting presented by two electrodes in a long narrow tube (Rompe and Schoen, 1938 a and b: Weizel, Rompe and Schoen, 1939). This tube may be, for example, 500 mm. long and 35 mm. in diameter, filled with 2 mm. argon pressure and a drop of mercury, and provided with electrodes of the type described above. A gas pressure of 2 mm. has been found to be more favourable in the experiments now to be described than the pressure of 9 mm. used in the experiments which have been described above.

This lamp starts, with electrodes initially cold, at about 400 volts, and has a running voltage of 100 volts when a current of 250 mA. flows. Experiment shows that sometimes the start of the glow discharge occurs in jerks followed by irregular dark periods.

When, however, a spiral wire is wound round the tube or an electrode of the type shown in figure 4 applied to the outside of the tube, and a high-frequency discharge (a few million cycles per second) of about 300 volts and a few mA. is

Figure 4.

connected to the outer electrode, while the second electrode of the high-frequency is connected to earth, the discharge starts with about 200 volts, and with electrodes of the above description, and resistance limiting the current to 250 mA., the change-over to an arc takes place regularly (Pirani, 1931).

What happens when the high frequency is connected to the spiral is that a glow discharge starts from the outer electrode to earth through the tube and the

electrode wires, which are both earthed through capacities of the order of 10 to 50 μF. This is immediately followed by the change-over to an arc on the electrodes.

It has been found experimentally that all methods by which a long discharge tube can be started successfully act in essentially the same way ; they favour the start of the glow discharge, which is seriously hampered in a long tube by negative wall charges which are probably generated by electrons diffusing towards the walls and which, according to the varying surface conditions of the inner glass wall of the tube, may be collected in irregular patches (Zeleny, 1920, 1922, 1938 ; Katayama, 1938).

The perfect starting system is therefore designed to remove all these wall charges effectively, and continuously to prevent their re-formation, which takes place at a considerable speed, i.e. within a few periods at 50 cycles per second.

Whilst high frequency is undoubtedly a very efficient way to do this, it is mostly undesirable from a practical standpoint because it requires expensive and complicated auxiliary apparatus. Therefore several other methods for removing the wall charges have been investigated, and one of them, while not quite as effective as the high frequency, may be briefly described (A. Bloch, M. Pirani and D. G. Prinz, 1939).

In this method, the external electrode of figure 4 is connected to earth through a high resistance of 2 to 4 megohms. It is difficult to realize why this should improve the starting. The oscillograph, however, reveals that surges are produced, but it is not known how these originate. One suggestion is that cosmic rays may be the cause. These seem to have the same effect as about 100 α particles falling on 1 cm^2 (McLennan *et al.*, 1902 ; Regener, 1933). The investigation showed that two kinds of surges are produced, having wave forms as shown in figures 5 and 6. The regular surges, shown in figure 5, are small

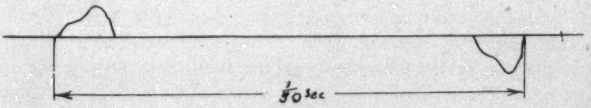

Figure 5.

and have no effect on the starting. The voltage kicks of figure 6, which represent a value of about 300 volts and occur every few seconds, seem to be those which neutralize the wall charges. They are difficult to record owing to their excessive

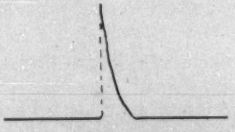

Figure 6.

speed, and in consequence figure 6 gives only their general appearance. The order of magnitude of the effective current, measured as an average on the outer electrode, is 10^{-6} amp. for a 35-mm. tube of 500 mm. length.

A comparison test made with a hairpin-shaped neon tube of 300 mm. length and 15 mm. diameter, which could be started with 500 volts without any starting device (having activated electrodes), showed that while only (245 ± 5) volts were needed with high-frequency excitation on the outer electrode, 300 volts were needed with the outer electrode connected in the way which has just been described.

A research was therefore conducted in order to find another method which would not require any outside electrode and which should have a result equivalent to the high frequency. This was found by using the following design, the aim of which was to generate a continuous stream of ions inside the whole tube so as to abolish the wall charges (Pirani, 1940). Emphasis must be laid on the fact that the whole of the tube is supplied with ions; therefore all such devices which only create a discharge in single areas of the tube (like the orthodox auxiliary electrodes, three types of which are shown in figure 7) are not suitable for long tubes (Uyterhoeven, 1938).

With this kind of device it very often happens that one or several separate discharges are started within the tube which nevertheless does not strike, obviously

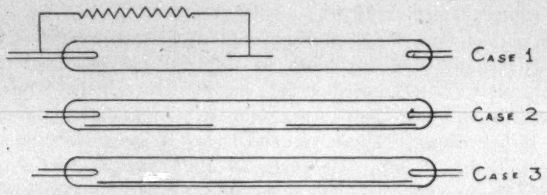

Figure 7.

because in one or several spots between these discharges there are still patches of electrons acting as negative grids. The mere presence of those patches indicates that the auxiliary electrodes do not fulfil their purpose, because they do not prevent the formation of islands of electrons through diffusion to the wall. This can only be achieved by means of a design which provides that ions are present at all times and in all parts of the tube, and especially before the striking voltage of the main discharge is reached; this means that the auxiliary discharge should work with a small phase difference from the main discharge and preferably have a lower starting voltage (hence the use of argon at 2 mm. pressure instead of 9 mm., which would have a lower starting voltage). The design which was eventually developed, after a considerable number of experiments, is given schematically in figure 8. It consists of two parallel wires forming a pair of electrodes extended through the whole tube, the wires being connected through small condensers to the electrode. The oscillogram of the tube, of 35 mm. diameter and 500 mm. length, fitted with condensers of 1 μF. connected between the main electrodes and the discharge wires, is shown in figure 9. Curve 3 is the voltage of the mains, curve 2 the voltage of the tube between the main electrodes H_1 and H_2, 1 the voltage between the auxiliary electrodes (which are connected through 1μF. to the main electrodes), 4 and 5 are the voltage

curves of the auxiliary electrodes and the main electrodes, to which they are connected through the condensers.

Whilst the accuracy of this oscillogram, so far as the phase differences between

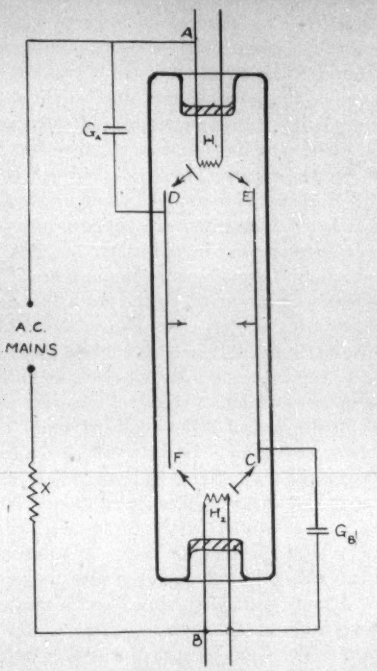

Figure 8.

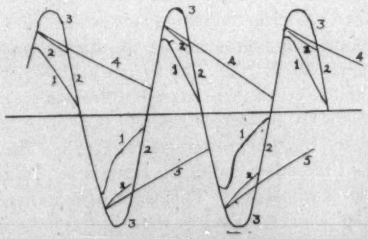

Figure 9. Voltage curves.

1, between the auxiliary electrodes C and D ; 2, between H_1 and H_2 ; 3, mains voltage ; 4 and 5, between AD and BC with 1 μF.

the different oscillograms are concerned, is not satisfactory (only one beam oscillogram was used), two things can be clearly seen. First, the striking voltage between the auxiliaries is lower than the striking voltage of the main discharge,

so that the whole of the tube is ionized and the walls are free from electrons. Of course a sufficient number of ions must strike the wall, but this is not a difficult task because the ionization of the glow discharge (with a current of about 0·02 amp.) takes place perpendicular to the tube axis. There is, however, another function which the auxiliary glow-discharge wires fulfil, namely, the upkeep of a voltage residue between each of them and the main electrodes, when the mains voltage goes through zero.

This means that a continuous glow discharge is kept alive which assists the change-over (with the possible help of metastable states) from glow to arc discharge between the main electrodes by ionic impact. It may be added that as a safety measure 0·01 gram of U_3O_8 was added to some of the pellets. (This is a radioactive compound which was found to produce about 10^4 ions per second.) However, it could not be proved that there was a real improvement, as this is a statistical problem which requires an extensive investigation.

The actual distribution of the amount of ionization current in the different parts of the tube depends on its length, on the gas filling and on the size of the condensers used to connect the main electrodes to the auxiliary wires. These can be chosen between 0·01 and 1 μF., and it is likely that in practice one could dispense with one of the condensers, leaving one electrode connected only through the natural capacity to the electrode. This is of the order of 10^{-5} μF. There is an additional means for the control of the ion distribution, that is the activation of part of the auxiliary electrodes. It was found practicable to activate the centre part by painting it with a solution of 99 parts of BaN_6 and one part of CsN_3. This activation paint should stop at a distance of about 5 inches from the main electrodes, from which the ends of the wires should be about 2 inches.

The next figures (10 and 11) show the design of an actual arrangement in a 500-mm. discharge lamp filled with 2 mm. argon and a drop of mercury. The starting voltage of this lamp was 200 V. when a series resistance consisting of a tungsten-filament lamp limiting the current to 250 mA. was used. Its running voltage was then 100 V. The lamp consists of a glass tube (4) which runs along its whole length. The tube is supported by an iron tube (5) which in turn is supported by the nickel bracket (3); this also supports the protective sheath (6). The stem (1), to which the nickel bracket (3) is attached, is provided with four lead-in wires. Two of these wires supply the filament (2), whilst a third is connected to the metal sheath (6). The fourth lead-in (7) is connected to an auxiliary electrode, and to reach this electrode it travels up through the tube (4) until it reaches the point (10). Here it emerges through a hole in the glass tube and is then wound round the tube, forming a spiral (8). The end of the auxiliary electrode coming from the other end of the lamp is shown as the spiral (9), and it is attached to a thicker wire at 11. This thick wire is anchored to the glass by a glass bead. The spirals are wound so that they are about 5 to 6 mm. apart. The glass tube is about 5 mm. in diameter. The arrangement does not suffer from the fact that the auxiliary electrodes are mounted eccentrically with respect to the stem ; on the contrary, it leaves the path of the main discharge free from all obstacles. As a matter of interest it may be added that this lamp started safely with 120 volts A.C. when the tungsten spirals were heated to a temperature of about 1200° C. by a small filament transformer.

§ 4. ACKNOWLEDGEMENTS

I have to thank Messrs. R. H. and H. N. Kirkness, who assisted me in the experiments at different times, for their efficient co-operation.

I am further indebted to my former colleagues, including Dr. C. C. Paterson, in the Research Laboratories of the General Electric Company at Wembley, and especially to Dr. N. R. Campbell, for many stimulating discussions about the electronic and ionic aspects of the problems dealt with in this paper.

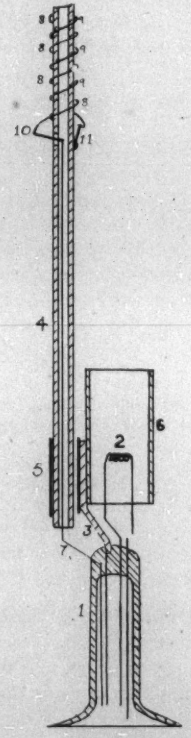

Figure 10.

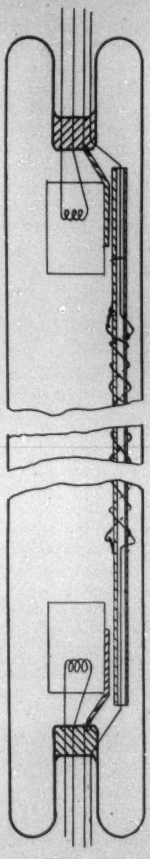

Figure 11.

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A NEW TYPE OF MICROPHOTOMETER

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Edinburgh

MS. received 29 *May* 1942

ABSTRACT. A new type of microphotometer is described which shows the blackening curve corresponding to the distribution of blackening along a straight line on a photographic plate, film or similar object instantaneously on the fluorescent screen of a cathode-ray oscillograph. The curve can either be measured directly on the screen or photographed for later use. A few examples of such photographic records are given as evidence for the proper functioning of the instrument. It could probably also be used for producing the intensity curve in a spectrum or the intensity distribution in X-ray scattering experiments.

§ 1. INTRODUCTION

THE investigation of the distribution of blackening in a photographic plate or film with the help of a microphotometer is so important for a great many branches of research work that such an instrument should belong to the normal outfit of every scientific laboratory dealing with such problems (e.g. physical, chemical or technical laboratories, observatories, etc.). Unfortunately existing instruments of this kind, although developed to a marvellous degree of technical perfection, and usually giving automatic records, are all very expensive and difficult to move from one place to another, whilst most of them require considerable skill and care in setting up and handling. Furthermore, the recording of the blackening curve along a straight line a few millimetres in length usually takes about ten minutes, and the survey of the blackening distribution in a two-dimensional plate of even a small area therefore becomes very tedious.

The new instrument which is described in the present paper is still only in an experimental stage and needs further technical development; but since the results hitherto achieved are already quite satisfactory, the publication of these results seems advisable. The main feature of the new device is that it gives a practically instantaneous record of the blackening curve (actual time of recording 1/100 second) which can be seen on a fluorescent screen, and consequently permits a quick visual survey of a whole plate. Further, it is possible to make a large number of records of those parts of such curves which are of special interest for the investigator, either by tracing them or by photography, in a time very short indeed compared with the time it would take to record these curves with the usual type of microphotometer.

The new instrument consists of two separate parts, the "receiver" and the "recorder", the latter being a cathode-ray oscillograph of the usual commercial type. Both parts are comparatively small and easily transferable. They are connected only electrically and avoid completely the use of any delicate mechanical device like exact worm-gearings and similar parts used in all existing types of microphotometers. Setting up and handling of the new photometer is very easy, and will certainly become even simpler once it has been developed into a compact instrument. Since the receiver contains no expensive parts and can actually be put together in any laboratory workshop from parts of existing apparatus, and since a cathode-ray oscillograph will probably be found in any medium-sized laboratory, the installation of the new microphotometer is very cheap compared with that of existing types.

§2. DESCRIPTION OF INSTRUMENT

In the following I shall give a short description of the principles of the new microphotometer illustrated by the schematic figure 1. The receiver consists of the light-source A, the microscopic arrangement B, the electromagnetic tuning-fork with the plate-holder C, and the photo-electric arrangement F. The recorder consists of the phase-shifter I, the amplifier G and the cathode-ray oscillograph H.

The light-source A consists of a 12-volt, 6-watt car lamp *a*, fitted into one end of a metal cylinder *b*, the other end of which is closed by a lens *c* for concentrating the light on the slit *f* of the microscope tube *g*. The current for the lamp is provided by a 20-v. battery of large accumulator cells through the rheostat *d* and the switch *e* for controlling the light intensity. The optical axis of the system A is horizontal and coincides with the optical axis of the microscopic arrangement B, which consists of a microscope of normal type (not shown in the figure) with its tube *g* set horizontally. The eyepiece of the microscope is replaced by a horizontal adjustable slit *f*. The whole of the slit is illuminated as uniformly as possible by the light from A. The objective *h* used is a Bausch and Lomb f = 16 mm., N.A. = 0·25, forming an image (1/12 scale) of the slit at a distance of about 5 mm. from its frontal lens. The condenser of the microscope was replaced by a suitable lens system *i* in order to concentrate the light finally on the cathode of the photo-electric cell.

The electromagnetic tuning fork consisted of an unmagnetized steel fork *j* tuned to a natural frequency of 100 c./sec., and a coil *k* with an iron core *l* placed

between the prongs of the fork and carrying an alternating current of 50 c./sec. By fixing two magnetized steel bars *m* to the prongs by means of iron clamps, thus polarizing the fork and at the same time reducing its natural frequency to about 50 c./sec., this instrument was transformed into a device for producing a practically linear and strictly harmonic vibrational motion with exactly the same frequency as the electric current used to operate it. The fork is fixed in a horizontal position on a support *n* fitted with a micrometric screw, so that, by turning this screw, it can be shifted vertically over a range of about 1 cm. The upper prong of the fork carries the plate holder *o*, made of aluminium sheet, into which a small plate E or film or a similar object can be fixed in such a way that it oscillates in a vertical direction with its plane normal to the optical axis of the optical system. The image of the slit is focused on the plate by means of the micrometer screw of the microscope.

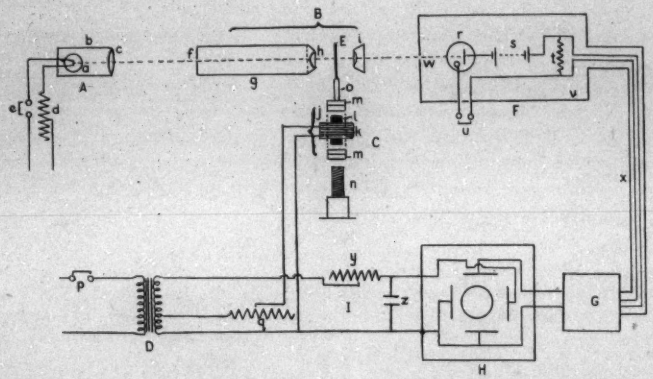

Figure 1.

The current for the fork is supplied from the 12-v. terminals of the secondary coil of a stepdown transformer D, the primary coil of which is connected through the switch *p* to the main 240-v. A.C. supply. The current, and hence the amplitude of the vibration of E, can be controlled by the rheostat *q*. Another means for altering the amplitude of the vibration, and at the same time for compensating for the change in mass of different plates, consists of two brass clamps fixed to the two prongs of the fork; by changing the position of these clamps, the natural frequency of the fork can be slightly changed and, therefore, the amplitude of its forced vibration altered.

The photo-electric arrangement F consists of a gas-filled photo-cell *r* with caesium-coated cathode, requiring about 100 volts, which is supplied by a high-tension dry battery *s*. The circuit contains a resistance *t* of one megohm in series with the photo-cell and a key *u*. Photo-cell, battery and resistance are enclosed in a tin box *v* acting as a shield against external disturbances, and provided with a slit *w* through which the light emerging from the lens *i* passes to the cathode of the photo-cell. The use of caesium as photo-electric agent

simplifies the optical part of the microscopic arrangement considerably, since the maximum sensitivity of the photo-electric effect of caesium lies in the visible part of the spectrum, and therefore all lens-systems corrected in the usual way for visible light can be used, whereas, for example, the microphotometers of the Moll type, which use a thermo-couple as detector, require optical systems specially corrected for infra-red light. The fact, on the other hand, that in the new instrument the recording is practically instantaneous renders superfluous all the complicated appliances needed in the existing types of microphotometer which use photo-cells as detectors (e.g. the Koch photometer) for keeping the zero of the recording electrometer or galvanometer constant during the period of recording. The same simplification applies to the demands on the constancy of the light source; whereas in all the existing microphotometers of the usual type great care must be taken to keep the light source absolutely constant at least over the period of recording, in the new instrument it is quite sufficient if a change in the intensity of the light source (and also in the electrical conditions of the photo-cell) is negligibly small in the time necessary for tracing or photographing the curve shown on the screen of the cathode-ray oscillograph, i.e. a few seconds.

Two leads, shielded by a metal tube x, connect the terminals of the resistance t with the input terminals of the resistance-coupled two-stage amplifier G, which provides a linear amplification of the changes in the potential difference across the resistance, sufficiently high for an adequate deflection of the beam of the cathode-ray oscillograph H, the y-plates of which are connected to the output of the amplifier. In the present arrangement, a Mitcham cathode-ray oscillograph (Type GM 3152) is used in which an amplifier of the kind just mentioned is incorporated.

The 50-volt terminals of the secondary coil of the transformer D are connected to the phase-shifter I, consisting of a variable resistance y (maximum value 0·25 megohm) and a condenser z of 0·1 mf. in series. The x-plates of the oscillograph are connected in parallel to this condenser, thus supplying them with a sinusoidal potential difference of the same frequency as the mechanical oscillation of the plate E. By changing the resistance y, the phase difference between these two oscillations can be changed and, in particular, be made zero.

§ 3. USE OF THE INSTRUMENT

The function of the instrument can now be easily understood. Let us suppose that the plate contains a photograph of a spectrum stretching in the vertical direction, so that spectral lines, bands, etc., are strictly parallel to the slit f. The vibration of the plate across the light beam is equivalent to a vibration of the slit image along the spectrum over a range equal to twice the amplitude of the vibration. Since, as explained above, the potential difference across the x-plates of the oscillograph performs an oscillation of the same period, and the deflection of the electron beam is proportional to this potential difference and has practically no inertia, the trace of the beam on the fluorescent screen performs a vibration in the horizontal direction with the same frequency and, if the phase shifter is adjusted correctly, also in the same phase as the vibration of the plate. Therefore the motion of the spot on the screen is an enlarged copy of the motion of the light beam along the spectrum. In the present arrangement the total

sweep of the cathode-ray spot on the screen is about 5 cm.; since the range covered on the spectrum can be varied between, say, 0·1 mm. and 2·0 mm., the magnification factor in the horizontal dimension ranges between 25 and 500.

The deflection of the beam in the vertical direction is proportional to the potential difference between the y-plates or, provided that one remains inside the range of linear amplification of the amplifier, also proportional to the potential difference across the resistance t, and, therefore, finally proportional to the intensity of illumination on the cathode of the photo-cell. But since this again is proportional to the transparency of the plate on the particular spot just passing the slit image, the spot of the oscillograph traces the blackening curve automatically, i.e. the same curve as is traced on the recording paper of a micro-photometer of the usual type. But since in our case this process is performed once in one-hundredth of a second and is periodically repeated, one has the impression of seeing a part of this curve steady on the screen; turning the support screw n makes the curve shift in a horizontal direction so that its different parts come successively into the field of observation.

The scale of the curve in the x-direction can, as explained above, be changed arbitrarily between certain limits. In order to determine the magnification factor, one has only to shift the curve so that either the right-hand end-point of the curve on the screen is transformed into the left-hand end-point or vice versa: the reading of the displacement of the support on its micrometer screw and the extent of the horizontal sweep on the screen immediately give the x-scale.

The scale in the y-direction can be arbitrarily changed within certain limits either by changing the intensity of the light source by means of the rheostat d, or by changing the width of the slit f, or, if possible, by changing the gain of the amplifier G. It is therefore very easy to adapt the instrument quickly to the given gradation and average transparency of the plate.

In order to use the curve for measuring the transparency, the end-points of the scale in the y-direction corresponding to complete transparency and zero transparency (or complete blackening) have to be marked in some way. This is done automatically if the part of the plate under observation is known to contain completely black and completely transparent points. If this is not the case, the deflections of the cathode-ray beam corresponding to zero and maximum illumination have to be determined separately. In spectroscopic work it is further desirable to derive the intensity in the original spectrum from the blackening curve. This can be done in the usual way by impressing intensity scales for different parts of the spectrum upon the same plate. These can be recorded separately in the same way as the spectrum itself and used for reducing the transparency (or blackening) scale into an intensity scale.*

It is also possible to investigate a particular part of a spectrum in greater detail by increasing the magnification factor in the x-direction or in the y-direction, after a rough survey of the whole of the plate, within a few seconds.

In order to obtain a steady picture on the screen, the motion of the fork and the plate must of course be stationary. After having switched on the trans-

* In this case care must be taken to keep the optical and electrical conditions of the instrument constant during the whole of this procedure. The experiments described in § 4 have shown that this condition can be satisfied without difficulty.

former circuit or after having changed the resistance q or the position of the compensating masses on the fork, one has to wait for about 10 or 15 seconds until the vibration has become stationary. During this period one will observe two curves instead of only one on the screen, since the phase difference between the electrical and the mechanical oscillation varies until the phenomenon becomes stable. Should there still remain two curves after stabilization has been reached, these two curves can easily be made to coincide by an alteration of the resistance y. If the mains voltage or frequency is not constant, the amplitude of the mechanical vibration will fluctuate slightly, which causes a corresponding slight fluctuation of the trace on the screen of the oscillogaph and a fluctuating splitting up of the curve into two parallel curves. I did not succeed in completely removing this disturbing phenomenon, which sometimes, if the fluctuations in the mains supply are strong, makes photography of the trace difficult. But it seems almost certain that, once the instrument has been further developed technically, this difficulty will be overcome also.

§ 4. BEHAVIOUR OF THE INSTRUMENT

In order to give some evidence on the function of the instrument, two objects were investigated and the traces on the screen recorded photographically. The photographs were taken with a Zeiss Contax camera with Sonnar objective 1 : 2, f = 5 cm. on Kodak panchromatic cinematographic film. The exposure time was 1/2 sec. Prints were made from these negatives by means of a Leitz enlarging apparatus on Kodak BG4 contrast bromide paper and ten times linear enlargement. The scale of these positives is almost exactly one-half of the scale on the screen of the oscillograph.

The first object was the photograph of a band-spectrum on a plate, a part of which is shown in positive, five times enlarged, as figure 6 of plate 1. The region indicated below this figure was investigated in our instrument in nine parts, overlapping each other, so that a continuous blackening curve could be easily put together from the single photographs.* This curve is shown in figure 1 of plate 1, and by comparing it with figure 6 one can, without difficulty, recognize the different lines and bands in the blackening curve. The scale of the curve in the horizontal direction is about 30 times the scale in the original spectrum. Remembering that the scale on the screen is about twice the scale on the recording photograph, one sees that the magnification factor in this case has the value 60. Three parts of the recorded region of the spectrum were separately investigated for more detail by using a smaller amplitude for the vibration of the plate. The corresponding curves are shown in figures 2, 3 and 4 of plate 1, and the three parts are indicated above figure 6. The scale of these photographs is about 70 times the scale of the spectrum, so that in this case the magnification factor is about 140, considerably larger than the magnification used in any existing type

* It is not proposed by the author that the new instrument should be used in this way in future investigations. The curve shown in figure 1 of plate 1 could certainly be recorded much more quickly with the usual kind of microphotometer than with the new instrument in the way described above because of the time necessary to make the enlargements and to put the separate parts together. As stated in § 1, the main purpose of the new instrument is a quick visual survey of a whole plate and the photographic recording of the blackening distribution in small parts of it which seem to be important for the investigation.

of microphotometer. Figure 5 of plate 1 finally shows another part of figure 1 on about the same scale in the horizontal direction, but with a larger scale in the vertical direction, achieved by increasing the intensity of the light source.

The second object was a microscopic scale consisting of parallel lines about 0·2 mm. length and 0·1 mm. apart on a circular glass disk. The disk was covered with black paper, leaving uncovered only a slit with a width equal to the length of the lines. It was fixed into the plate-holder instead of the plate E, with the lines strictly parallel to the slit *f*. A number of photographs was taken with different amplitudes of vibration and the same degree of illumination. These photographs are shown on plate 2. The peaks of the curves correspond to the lines of the scale, and their distance therefore corresponds to a distance of 0·1 mm. on the original object. The scales on the five photographs are consequently 190, 130, 90, 60 and 40 times the original scale, and the magnification factors of the trace on the screen along the curves are about 380, 260, 180, 120 and 80, far exceeding the possibilities of magnification of the usual microphotometers. The object itself had purposely not been cleaned thoroughly, so that the transparency of the background should not be uniform. This is revealed in the records, especially number 5, by the deviation of the bottom part of the curves from straight lines.

Although the curves reproduced in plates 1 and 2 are quite satisfactory, it must be admitted that they perhaps show less detail than is normally shown in a curve recorded by a microphotometer of the usual type. This may be partly due to a slight inertia of the gas-filled photo-cell, and could be put right by using a vacuum cell together with a more powerful amplifier or a more powerful light-source. It may also be partly due to the comparatively great thickness of the curves as compared with the thickness of the curves obtained by other photometers. This thickness depends entirely on the size of the spot of the cathode ray on the fluorescent screen, and can be considerably reduced by reducing its brightness. But as in doing this I would have been compelled to increase the exposure time, and hence increase the probability of fluctuations during exposure, I have abstained from it.

It is, of course, possible to use the new instrument not only for the investigation of spectra or similar objects where the transparency varies in one direction only, but also for the investigation of plates or films with an arbitrary distribution of transparency, like photographs of x-ray scattering in crystals or any other photographs showing a delicate and complicated two-dimensional pattern. In this case, one has only to replace the slit *f* by a narrow circular aperture and to provide the instrument with a second device, similar to the support *n*, for shifting the plate E in a horizontal direction in its own plane.

It is further possible to use a similar instrument not for the investigation of the blackening distribution in the photograph of a spectrum but for the investigation of the intensity distribution in the spectrum itself. Such a device is very useful for many branches of spectroscopy, e.g. in astrophysics, and recording instruments for this purpose, based on the principles of the usual microphotometers, exist already. In order to use the principle of the present instrument for the same purpose, one can, for instance, replace the light source A by the spectrometer in such a way that the part of the spectrum to be investigated is

Figure 1. ~ 1 : 30

Figure 2. ~ 1 : 70

Figure 3. ~ 1 : 70

Figure 4. ~ 1 : 70

Figure 5. ~ 1 : 30

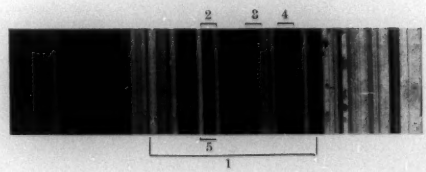

Figure 6. Original spectrum, 5 times enlarged.

Plate 1.

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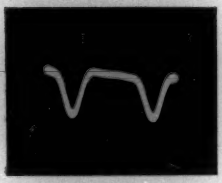

Figure 7. ~ 1 : 190

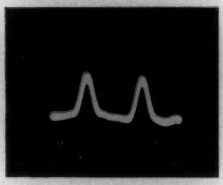

Figure 8. ~ 1 : 130

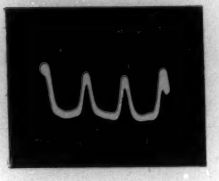

Figure 9. ~ 1 : 90

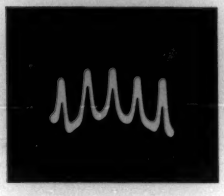

Figure 10. ~ 1 : 60

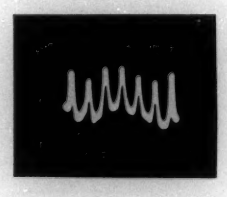

Figure 11. ~ 1 : 40

Microscopic scale, one division = 0·1 mm.

Plate 2.

projected on the widely opened slit f; the plate E has then to be replaced by an opaque plate with a very narrow horizontal slit. When this slit moves up and down, it allows the passage of light from the different parts of the spectrum to the photo-cell, and one sees immediately that this must produce a trace of the cathode ray on the screen of the oscillograph in the form of a curve plotting the intensity in the spectrum against the position along the spectrum (provided that the sensitivity of the photo-cell does not vary within the region covered by the oscillation of E).

A similar arrangement will probably be applicable for the direct investigation of the intensity distribution in an X-ray spectrum or the X-ray scattering by material bodies, especially crystals. In this case one has only to replace the photo-cell by an ionization chamber or similar device for transforming the variations in X-ray intensity into variations of an electric current. This idea was suggested by Professor M. Born. An instrument of this kind would certainly be very useful for many problems, e.g. the so-called background scattering by crystals, and would yield material for a systematic investigation of the dynamics of crystals.

§ 5. ACKNOWLEDGEMENTS

I wish to express my sincerest thanks to the Royal Society for a research grant; to Professor Barkla, of the Physics Department of this University, for allowing me to use the resources of his department; and to Professor Ritchie, Zoological Department, Professor Greaves, Royal Observatory, and Dr. Childs and Dr. Milne, Physics Department, for lending me various instruments and appliances; and last, but not least, to Professor Born, Mathematical Physics Department, for his great interest in my work and for his encouragement.

THE VISCOSITY TONOMETER—A NEW METHOD OF MEASURING TENSION IN LIQUIDS

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ABSTRACT. A new method of measuring hydrostatic tension within a liquid is described. The gas-free liquid is enclosed in a glass bulb to which is connected a fine capillary tube. By controlled cooling of the bulb, tension is created which pulls the column of liquid down the capillary tube. The highest rate of flow which does not cause a break in the liquid is observed, and the corresponding tension calculated.

§ 1. INTRODUCTION

It was desired to investigate, by some means independent of the elasticity of the liquid or of the containing vessel, the hydrostatic tension which can be established inside a glass vessel by controlled cooling of the liquid. The method adopted has been termed the *Viscosity-Tonometer* method, and will now be described in detail.

§ 2. THE VISCOSITY TONOMETER

The term *tonometer* seems to have been first used in its present sense by Worthington (1892). This meaning is given in the Shorter Oxford Dictionary as "an instrument for measuring strains in liquids". It is clear that what is meant is "an instrument for measuring stresses in liquids". One meaning of the word *Tonometry* is given as the "measurement of tension".

The tonometer is illustrated in figure 1 and is constructed entirely of soda glass. The lower part consists of a long tubular bulb, B, to the upper end of which is joined a fine capillary tube, T. The top end of the capillary tube is joined to a wider tube, R, about 2 mm. internal diameter. Having filled the

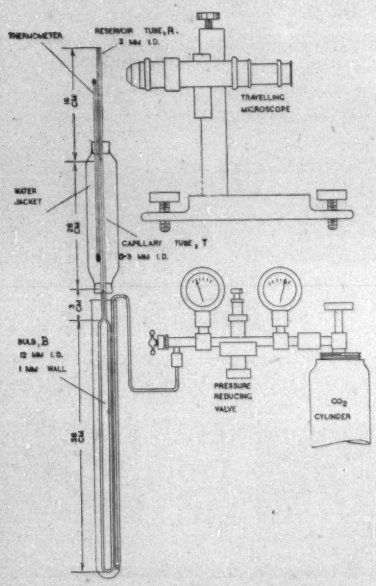

Figure 1.

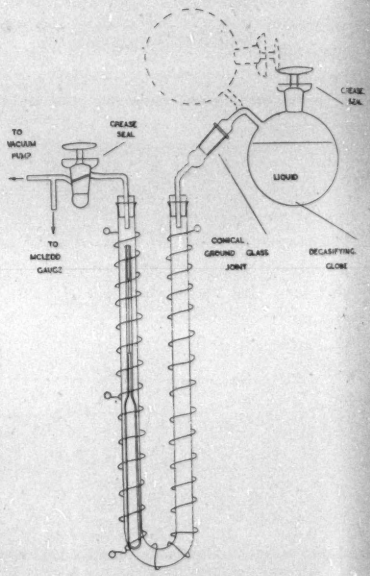

Figure 2.

instrument with the liquid, taking precautions to ensure that both the liquid and enclosure are as free from gas as possible, the temperature of the bulb is lowered, causing the liquid therein to contract and more liquid to flow down from the reservoir tube. This flow is due partly to the atmospheric pressure, and partly to the hydrostatic tension within the liquid. The rate of flow is determined by observing the position of the meniscus at intervals of 30 sec. by means of a microscope cathetometer fitted with a vernier reading to 0·001 cm. The highest rate of flow which can be produced without breaking the liquid at any point is observed. The rate of flow after a break has occurred is also observed, this flow being entirely due to atmospheric pressure, provided that the pressure n the bubble formed is negligible.

From the ratio between the rates of flow taken before and after the break, the tension in the bulb can be calculated. During the work, the pressure above the liquid in the reservoir tube is normally at atmospheric, but can be lowered to about 0·1 mm. Hg by means of a vacuum pump ; thus any gas liberated when a break occurs is readily removed from the system in readiness for another attempt. The validity of the viscosity-tonometer method rests on the assumption that the flow through the capillary tube is proportional to the difference of pressure between the ends. This must be so for any liquid which does not exhibit anomalous viscous properties provided that the rate of flow is never high enough to produce turbulent motion, and provided that the viscosity is not appreciably affected by the state of tension which exists in part of the liquid column.

Since we are concerned only with orderly flow, the method is valid for any cross-section of bore in the capillary tube. For example, the bore could be oval, as it usually is in thermometer tubing. A more important advantage is that a glass filament may be inserted in the tube to increase the resistance to flow for measurements on liquids of low viscosity.

A viscosity tonometer having been constructed, it was placed in one arm of the glass U-tube shown in figure 2. The internal diameter of the tube was just large enough to give a convenient diametral clearance of about 3 mm. Mineral oil as used previously in the bellows experiments (Vincent, 1941) was again employed in the present work. It was of the paraffin base type and described by the makers as triple filtered. At 20° c. it had a viscosity of 16·5 poise and a density of 0·893 gm. per c.c. A quantity of this oil was thoroughly degasified by shaking and heating in the glass globe under an absolute pressure of approximately 0·1 mm. Hg. During the degasification process, the tonometer and U-tube were heated while under vacuum by passing a current through the heating coil. After thorough degasification, the oil was poured into the empty arm of the U-tube by turning the globe into the position shown dotted in figure 2, and the absolute pressure of about 0·1 mm. was maintained until the oil level reached the vacuum connection, when the glass cock was closed and the stopper removed from the neck of the globe to apply atmospheric pressure and thus drive the oil into the tonometer. To reduce the solution of air in the oil during this filling process, the heating current was applied only to that part of the coil surrounding the narrow part of the tonometer. The whole apparatus was left like this overnight, and the next morning the tonometer was found to be completely filled with oil. The tonometer was removed from the filling apparatus and set up with the reservoir tube clamped in front of the object glass of the cathetometer. A water jacket was provided round the capillary tube, as illustrated in figure 1. This served to maintain a steady temperature around the capillary tube, and provided a place for insertion of a thermometer, which was, however, required for record purposes only. The cylindrical bulb of the tonometer was provided with a cooling jacket. This consisted of a wide glass tube sealed at the lower end. CO_2 from a cylinder was supplied through a diaphragm-operated pressure-reducing valve and fed in a slow stream by means of a glass tube to the bottom of the cooling jacket. This system of cooling was found to be extremely satisfactory and easy to control, and provided a gradually

increasing rate of cooling. It is clear that the pressure regulator and delivery tube become cooled before the cooling effect of the CO_2 has much influence on the tonometer bulb. It will be seen later, on examining the results, that a gradual increase in the rate of cooling is advantageous, since, in conducting an experiment, the cooling rate is allowed to increase until the breaking tension is reached.

§ 3. EXPERIMENTAL RESULTS OBTAINED WITH THE VISCOSITY TONOMETER

The graph in figure 3 shows the cathetometer readings at 30-second intervals after commencement of the CO_2 flow. After the first three readings, the liquid was heard to break with a click. The temperature of the capillary tube at the moment of break was 20° C. A tangent drawn to the first branch of the graph

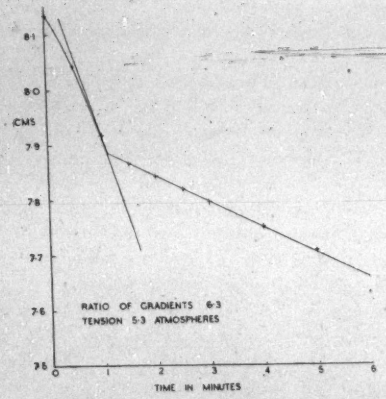

Figure 3. Viscosity tonometer (1st run).

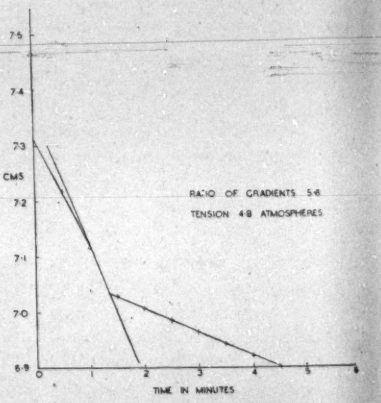

Figure 4. Viscosity tonometer (2nd run).

is a measure of the rate of flow immediately prior to the break, and the gradient of the second branch of the graph is a measure of the rate of flow immediately after the break. The temperature of the capillary tube and all other conditions are identical for the two rates of flow. The ratio of the gradients is therefore equal to the ratio of the pressure differences before and after the break. In the present case the ratio of the gradients is seen from the graph to be 6·3, so we have

$$\frac{\text{Atmospheric pressure + tension}}{\text{Atmospheric pressure}} = 6{\cdot}3\,;$$

therefore Tension = 5·3 atmospheres.

It will be noticed that this result is in terms of the atmospheric pressure actually obtaining at the time of the experiment. For very precise results this should be observed, and the results stated in terms of a standard atmosphere.

After obtaining the above results, the water jacket was removed from the

capillary tube and the tube gently heated with a Bunsen flame to reduce the viscosity of the filament of liquid therein. The tonometer globe was then gently heated in the same way, and the bubble formed from the previous break was expelled through the capillary tube. The temperature of the whole apparatus was then allowed to re-establish itself at approximately atmospheric temperature before carrying out the next set of readings, which are plotted in figure 4, from which it will be seen that the ratio of the gradients is 5·8, and hence the tension 4·8 atmospheres. After each test such as the above, a bubble was eliminated from the tonometer. Sometimes it was found convenient to apply vacuum to the top of the reservoir tube to assist in the removal of the bubble. The initial conditions were, however, approximately the same in all the tests carried out. The results of subsequent tests are set out in the table below:—

Table 1. Summary of results so far obtained with the viscosity tonometer

Run number	Breaking tension (atmospheres)	Temperature of capillary tube at moment of break (° c.)
1	5·3	20·0
2	4·8	20·6
3	2·6	19·3
4	2·6	20·6
5	1·6	21·1
6	2·7	22·5
7	3·6	23·3
8	7·8	23·6
9	5·7	23·2
10	6·8	17·7
11	4·8	18·9
12	6·3	22·1
13	1·7	23·0
14	5·9	23·5

Since the eighth run, in which the highest value of 7·8 atmospheres was obtained, is of special interest, these results are plotted in figure 5.

It will be noticed that the highest result obtained for mineral oil in the viscosity tonometer was 7·8 atmospheres, which is considerably higher than it was possible to obtain in the metal bellows, in which the highest value was 2·92 atmospheres. It thus appears that the use of a glass vessel does permit the application of higher tensions to the liquid, but it does not explain the remarkably high values given by the Berthelot method. The writer has found that, using the same mineral oil in the Berthelot method, values of tension up to 119 atmospheres are obtainable. Another point remaining to be investigated in connection with the Berthelot method was whether the high pressure it is possible to apply by heating above the sealing temperature permits the application of high tensions to the liquid during the subsequent cooling. It was thought quite possible that the effect of pressure might be to cause the liquid to penetrate

any surface cavities and thus reduce the size of the spaces available for the formation of unstable bubbles. The simplest way of examining the effect of high pressure was to place the whole tonometer inside an enclosure and subject it to a high gas pressure, both externally and on the column of liquid in the reservoir tube. This method has the objection that there is a risk of penetration of dissolved gas to the interior of the tonometer. However, it was thought that

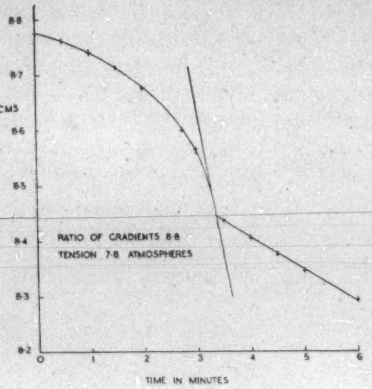

Figure 5. Viscosity tonometer (8th run).

this way of applying pressure would be satisfactory, since the rate of diffusion of dissolved gas through viscous oil at atmospheric temperature is not very great, and it so happens in the tonometer method that the greatest tension is at the lower end of the capillary tube, to which point it would take a considerable time for gas to penetrate.

§ 4. EFFECT ON TONOMETER READINGS OF PREVIOUS APPLICATION OF PRESSURE

A steel tube was prepared from hydraulic tubing. A plug was welded into the lower end and fittings were provided for connecting it to a cylinder of nitrogen. The tonometer was placed in the tube and exposed for a period of 68 hours to a nitrogen pressure initially at 120 atmospheres, which, due to leakage, fell gradually to 60 atmospheres. The pressure was then lowered gradually to 10 atmospheres over a period of 4 hours. The connections were then unscrewed and the tonometer removed. A run was then obtained under the same conditions as previously described.

The results were graphed as before and the ratio of the gradients was 7·6, giving a tension of 6·6 atmospheres.

It was decided to repeat this experiment, so the tonometer was replaced in the steel tube and nitrogen pressure applied for a period of 22 hours. The excess pressure was initially 57 atmospheres, which fell during the pressure period to 50 atmospheres, which was slowly released to 10 atmospheres before

removal of the tonometer, when results were obtained which, on plotting, gave a tension of 2·9 atmospheres. Eliminating the bubble after the above test, another value of 2·9 atmospheres was obtained, followed by a further result of 4·1 atmospheres. All the results obtained after the application of gas pressure are set out in the table below:—

Table 2. Results obtained with the viscosity tonometer after application of gas pressure

Run number	Breaking tension (atmospheres)	Temperature of capillary tube at moment of break (° c.)
15	6·6	22·1
16	2·9	21·4
17	2·9	22·3
18	4·1	21·1

§ 5. DISCUSSION OF RESULTS

Taking the highest result obtained with the viscosity tonometer, i.e., 7·8 atmospheres, and substituting in the equation $R=2S/T$, where S is the surface tension in dynes/cm. and T is the observed breaking tension in atmospheres, we have

$$R=\frac{2\times 30}{7\cdot 8\times 1\cdot 013\times 10^{6}}=7\cdot 6\times 10^{-6}\text{ cm.}$$

This is the radius of the largest bubble which could withstand a tension of 7·8 atmospheres. It seems reasonable to suppose that this radius is related to the dimensions of cavities in the glass surface which were unfilled by the liquid under tension. Comparison of this value of R with the corresponding value of $2\cdot 0\times 10^{-5}$ cm. obtained on the same oil in the bellows apparatus probably gives an indication of the difference in magnitude of the unfilled surface imperfections. The difference in bubble radius may also be partly due to differences of angle of contact with the sides of the unfilled cavities.

By comparison with table 1, the above results show that previous application of pressure to the liquid in the tonometer did not produce high values of tension. Since this application of pressure makes the conditions in the tonometer method and in the Berthelot method almost identical, the great difference in the general level of the results obtainable by the two methods clearly needs further investigation. In this connection it is interesting to compare the tonometer results in the present paper with the summary of results by the Berthelot method given previously (Vincent, 1941).

The viscosity-tonometer method has given results for mineral oil which are considerably higher than the highest value of 2·94 atmospheres obtained by the bellows method, and described previously (Vincent, 1941). This illustrates the important effect the nature of the enclosure has upon the tension it is possible to apply to liquids.

§ 6. ACKNOWLEDGEMENT

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TURBIDITY MEASUREMENT BY OPTICAL MEANS

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§ 1. GENERAL PRINCIPLES

WHEN a beam of light passes through a fluid containing particles in suspension, some of the light gets through while the rest is scattered, i.e., diverted in random directions from the path of the original beam. We shall pass over such phenomena as the Raman effect, in which light is literally absorbed by the molecules and re-radiated when they oscillate, since it will be assumed that the true turbidity effect outweighs any anomaly due to molecular vibration or absorption in the fluid.

The general law for the extinction of light (using this word to cover both scattering and literal absorption of the radiation as it passes through the suspension) is

$$I_x = I_0 e^{-kxc}$$

(Lambert-Beer law), where I_0 is the incident intensity, I_x that after passing through a length of path x, and c the concentration. This law is found to hold for light of a single colour provided the concentration of the suspension is not too great.

When the colour of the light is changed or if a mixed, e.g. white, source is used, then it is found that the extinction coefficient k varies with the wave-length of the light. In particular as the average size of particle approaches the wave-length of the light, this coefficient increases.

For particles considerably less in size than the wave-length, Rayleigh's law, that the extinction is inversely as the fourth power of the wave-length, holds (Rayleigh, 1871).

When the particles are comparatively few in number, and those large compared to the wave-length of the light, it is fairly obvious that the light cut off, e.g. from a photographic plate placed behind the tank containing the suspension, will be proportional to the total projected area of the particles, i.e. to $\Sigma n r^2$, where n is the

number of particles having a radius r. This has, in fact, been verified by using monodisperse suspensions, that is, those having particles all of one size. (Such suspensions can be obtained by laborious fractionation in the centrifuge or by using the spores of certain fungi which are found to be uniform in size.)

The criterion that decides when the influence of the wave-length of the light becomes important is $2\pi r/\lambda$. When this parameter works out to be less than 10, the amount of light cut off becomes greater than what one would expect if it were a simple shadow effect, the loss in transmission being made up by an increase in the light scattered to one side. The following table gives, for light of different wave-lengths, the particle diameter about which this abnormality becomes noticeable.

Light source, λ	Infra-red, $1\,\mu$	Red, $0{\cdot}6\,\mu$	Green, $0{\cdot}5\,\mu$	Violet, $0{\cdot}4\,\mu$	Ultra-violet, $0{\cdot}36\,\mu$
Diameter of particle (d)	$3{\cdot}2\,\mu$	$1{\cdot}9\,\mu$	$1{\cdot}6\,\mu$	$1{\cdot}3\,\mu$	$1{\cdot}2\,\mu$

The writer has conducted a research into this matter (Richardson, 1940), the results of which are presented in figure 1, together with a theoretical prediction (full line) of the effect due to Jobst (1925). In the experiments a titanium-oxide sol was the medium and an ultra-violet source (mercury-arc line $0{\cdot}365\,\mu$) was used, the amount of light passing through the sol being measured photo-electrically. The horizontal axis of the graph represents the absorption that one would get if it were truly proportional to the surface area of all the particles, the deviation being shown by the graph rising above this axis below $2\pi r/\lambda = 10$; $d = 1{\cdot}1\,\mu$.

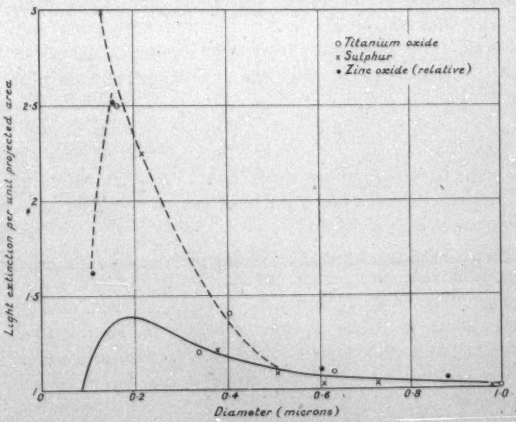


Figure 1. Variation of light-extinction with diameter.

Houghton (1942) has also dealt with the extinction and scattering of light by a conglomeration of small particles of uniform size. Stutz (1930) has used zinc-oxide suspensions showing the relative transmission at different values of d/λ. His results, though not calculated in terms of extinction per unit area, are included on figure 1, being made to agree with the sulphur value at $2\pi r/\lambda = 1{\cdot}4$.

No theory fits the experimental results over the whole range of size from $1\,\mu$ to $0{\cdot}1\,\mu$, though Jobst's curve suits the upper portion of the range fairly well. In any case a number of factors intervene in the theory, such as the electrical conductivity of the material and its dielectric constant or refractive index, to which it may be difficult to attach values appropriate to a given powder (Jobst's curve represents insulating particles). It must also be remembered that no matter how patiently one centrifuges, each experimental sample of supposedly uniform size splashes over into the adjacent lower and higher grades, whereas theory assumes absolute uniformity of sizing.

It is probable that each worker who desires to explore the range between 2 and $0{\cdot}2\,\mu$ will have to do a calibration, in some such way as this, on the material in which he is particularly interested. For instance, the experiments of Whytlaw-Gray and Patterson (1926) and of Cheveneau and Audubert (1920) on the optical properties of other types of suspensions do not agree with those quoted here. In working with a large number of materials and using white light, however, the writer has found that the "shadow" effect (extinction proportional to projected surface) holds with sufficient accuracy down to 2 or 3μ. (In an earlier paper it was suggested on the basis of Cheveneau and Audubert's work, but in advance of confirmatory experiments, that in the region 1 to $10\,\mu$ a linear law between extinction and *diameter* would hold; but the calibration showed that the conditions in the French scientists' work did not apply to the author's apparatus.)

In using a turbidimeter in which the wave-length of light bears this type of relation to the particle size, a correction to the readings of light extinction is required in the sense of these results. The application of this correction is discussed later, but it may be noted that the majority of suspensions met with industrially fall, as to size, above these limits. Exceptionally, certain colloidal clays and bacterial cultures which do come in this region need to be estimated in soil and microbiological science.

The effect of excessive concentration of the suspension on turbidity readings is in the opposite sense since, examining the problem in terms of shadowgraphy, the silhouettes of the individual particles overlap, giving a total light extinction less than that which would be expected if it were the sum of all the extinctions produced by isolated individuals. Fortunately, with photoelectric recording, it is an advantage to use a small concentration, so that this interference effect does not preponderate.

When measurement of turbidity *per se* is wanted, apart from an analysis of particle size, or of the specific surface (Σnr^2 per gm. in the above discussion), it may be preferable to measure the light scattered sideways from the suspension rather than that which passes out in the direction of the incident beam. The two methods are usually distinguished as ***nephelometry*** and ***turbidimetry*** respectively. The sideways view is to be preferred if the suspension is one of very fine particles in dilute concentration. A research by Fawcett and Hewitt (1939)

shows that, using fuller's earth suspended in water, the direct turbidimeter is the more accurate at larger concentrations, the nephelometer at low, with a change-over at about 4 parts in 100,000.

§ 2. TYPES OF TURBIDIMETER

A. *Visual*

In these methods a comparison is made by eye with a standard illuminant. If the light (transmitted or reflected) is coloured, this method allows of the tint of the standard being first adjusted to equality with the light under examination. Subsequent estimation of intensity is then a question of varying the amount coming from the standard. In the Lovibond-Schofield apparatus, the standard is the light reflected from a white block of magnesium carbonate after passing through a series of suitable colour filters. An obturator vane V (figure 2) pivoted between the white source (in the form of a glass screen W illuminated by daylight reflected from the mirror M) and the two samples A_1 and A_2 to be

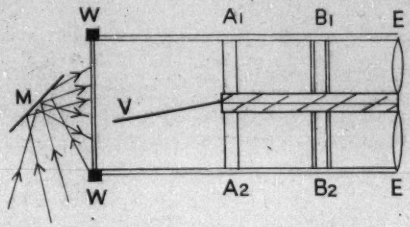


Figure 2. Diagram illustrating Lovibond-Schofield apparatus.

compared allows the relative brightness on either side to be varied as the vane is rotated. E represents the eyepiece through which the matching is made, B_1 and B_2 the filters required to make the tints the same. In turbidity work A_1 would contain the suspension and A_2 the pure liquid. A scale attached to V can be graduated to indicate turbidity in terms of concentration of fuller's earth or other standard. (In the figure only the *principle* of the instrument is represented. In practice, the optical arrangement is a reflecting instead of a transmitting one, but would be more difficult to follow without a demonstration on the actual instrument.)

B. *Photoelectric*

In this case there is no subjective comparison. The scattered or transmitted light falls on a photoelectric cell and the current produced therein is measured on a sensitive galvanometer to act as a measure of the light falling on the cell. Either a gas-filled or a rectifier (photronic) type of photoelectric cell is used. Unless the illumination is excessive, both types will give a current proportional to the light falling upon them. The former requires a polarizing voltage from a high-tension battery whose value is adjusted to give the requisite direct proportionality between intensity of illumination and current : the latter type may be directly connected to the galvanometer.

The procedure then is to pass the beam of light first through a glass trough containing the suspension and to note the current. The tank is next emptied and the suspension replaced with the (same) pure liquid and another reading taken. In some instruments the turbidity is not calculated in terms of the two photoelectric currents but by absorbing some of the light in a calibrated wedge of glass or by cutting off some of it with an adjustable stop until the second galvanometer reading is equal to the first. If the apparatus has been previously calibrated, e.g. by introducing artificial turbidity in terms of known concentrations of fuller's earth, it is possible to read turbidities directly on the dial which makes these adjustments. (This, of course, assumes a standard-size grading in the fuller's earth used for the calibration.)

It is to be noted that the photoelectric cell is sensitive to colour as well as to intensity of light. It is therefore important to see that the colour transmitted or scattered by the suspension has the same tint as that from the pure liquid.

§ 3. ESTIMATION OF PARTICLE SIZE

In this process we have a powder containing particles covering a range of size, and we want to know what mass of particles lies within certain limited particle-size groups. This is often known as *mechanical analysis*, a term due to the soil physicists. Particles greater than 120 μ are usually estimated by graded sieves, the remainder by using the law relating their rate of fall through a liquid to their respective diameters. A representative specimen of the powder is stirred

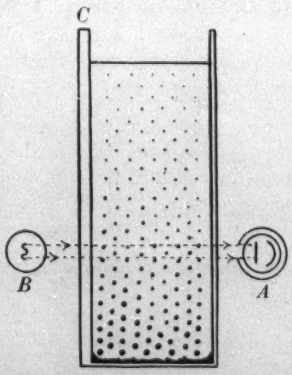


Figure 3. Diagram illustrating particle-size determination by photoelectric means.

up with water or other suitable liquid in a tall tank (figure 3) and the particles allowed to settle under gravity. The rate of settlement depends on the difference of density of solid and liquid, on the viscosity of the latter, and varies as the square of the particle diameter, i.e. as the superficial areas of the particles if they can be assumed all to have the same shape. Thus at a given depth, h, below the surface of the liquid, if samples are taken at various times after the initial dispersion of the powder, the early samples will be fairly representative of the whole suspension;

but as time goes on they will become progressively deficient in the larger particles which settle more quickly, until eventually samples will contain only those supported by molecular agitation and too small to sink. Thus if we take samples of soil in water at 17° c. at a depth of 30 cm., particles of 10 μ will all be past this point in 60 min., those of half this size in 240 min., whereas all smaller than 5 μ still remain in both samples with their original relative concentrations unchanged. Thus the difference in weight of the two samples at 60 and 240 min. represents a decrease due to all those between 10 and 5 μ, and so on for other size grades.

§ 4. PHOTO-SEDIMENTATION METHOD

A. *Constant depth*

The general principle of the method is to apply the usual sedimentation procedure, but to use such small concentrations of solid in liquid that the error of the dragging down of small particles by large ones and the consequent formation of aggregates is largely avoided. The sedimentation of the powder is followed by measuring the light which it cuts off at a horizontal plane (AB, figure 3) near the base of the tank, using a narrow beam of light and a photoelectric cell for this purpose (Richardson, 1933 and 1934).

The powder is first thoroughly dispersed in the liquid so as to give a homogeneous suspension and then the light-extinguishing power of the suspension is determined at the selected depth and at known intervals of time after the start of settling. The photoelectric current naturally rises during the course of an experiment owing to particles above a certain size having all got past the observation point after a given time. The time in question is related to the size by Stokes' law or one of its modern modifications. For particles smaller than 80 microns we may take it in its original form, i.e. $V = g(\rho - \sigma)d^2/18\eta$, where V = velocity of falling particle, d = diameter of particle, ρ = its density, and σ and η = the density and viscosity respectively of the liquid. For a given temperature the quantity $g(\rho - \sigma)/18\eta$ remains constant during an experiment, so that we may write $d = \sqrt{V}/C = (1/C)\sqrt{(h/t)}$, where C is constant, as the relation between the particle diameter and the time it takes to pass from the surface of the liquid to the level of the light beam, a distance h. Thus at any time t_1 since the commencement of sedimentation, the diameter of the particles of size d_1 which are just clearing the observation point may be calculated, these final members of this size being naturally the ones that had to start from the surface of the liquid. Similarly the particle size d_2 just clearing at a time t_2 may be calculated. Thus the rise in photoelectric current between the times t_1 and t_2 from the start may be ascribed to all those particles whose size lies between d_1 and d_2. Further, if we plot a curve of photoelectric current against time, any ordinate on this curve may be ascribed to the net effect of all particles smaller than the size d which corresponds to the time t over which the ordinate stands. It remains to determine the "specific" power of light-extinction which is to be ascribed to a particle. It has been shown by experiment that as long as the particle is large compared to the wave-length of the light used, its power of light-extinction is proportional to its projected surface, i.e. it acts as a simple obstacle to the light. In the case of a spherical particle, then, the geometrical projection of all the n particles of size d is $n\pi d^2/4$. When particles of size between d_1 and d_2 are

under consideration, the change in photoelectric current occurring during the above time interval is due to the total surface area of this size group of mean diameter $\bar{d}$. But the weight of the particles in this grade is proportional to the total volume, i.e. to $\bar{d}^3$, whereas the change in light current gives us a figure proportional to $\bar{d}^2$. It is therefore necessary to divide up the curve of current against time into appropriate steps, depending on the size grades selected, and to multiply the increase in current for each grade by the mean diameter in the grade. The figures so obtained give the relative weights in each size group.

The sedimentation is at present carried out in a cemented tank of sintered glass, 4 cm. square and either 10 or 15 cm. high. This is placed at a set position in a blackened wood box which is light-tight. The vessels thus hold 160 or 240 c.c. of liquid respectively, and the quantity of solid used is generally about 0·2 gm. Sometimes a dispersing agent such as ethylene glycol is added. The suspension is uniformly distributed at the start of the experiment either by stirring or, more effectively, by having a stopper to cover the top of the little tank and inverting it a few times before it is placed in position (figure 4). The light source is a low-power electric bulb focused by means of a cylindrical lens to form an image, 1 cm. broad and 2 mm. high, in the centre of the glass tank. On the other side of the latter is another cylindrical lens and the photoelectric cell (rectifier type) which is connected to a sensitive galvanometer (Tinsley or Cambridge "Spot"). The apparatus is shown in figure 4 *a*.

The liquids used depend on the specific gravity and mean size of the particles concerned. For sands and soils, water or mixtures of butyl phthalate and alcohol; for chalk, coal, etc., acetone or petroleum ether. Unless water at a known temperature is being used, the viscosity and density of the liquid must be determined. When a continuous analysis by sedimentation is made, shallow S-shaped curves of the type shown in figure 6 are obtained, such that any ordinate represents the "weight" in the statistical sense of all those particles larger in diameter than that value of the abscissa over which this ordinate stands. This is the summation curve of the specimen. The distribution curve, in the form of numbers corresponding to a given size, is found by drawing tangents to this first curve and plotting them against the same abscissae.

Since the viscosity of a liquid varies with its temperature, this must be measured and maintained constant during an experiment by enclosure of the tank in a thermostat or by circulating water in a larger tank closely surrounding the glass one; this precaution is also necessary to prevent convection currents spoiling the results.

B. *Constant-time method*

An alternative procedure, suitable when the particles fall quite slowly in the liquid, is to wait an hour or so until they have taken up a suitable distribution in the cell like that shown in figure 3, and then to take a series of readings as quickly as possible of photoelectric current at different levels through the whole tank, moving it down step by step on a ratchet movement past the stationary beam of light AB and so getting readings of light transmitted for a constant value of the time t, but various values, h_1, h_2, etc., of the depth below the surface. The curve resulting on plotting these is the usual summation curve from which

values of the velocity V, and hence the particle size d corresponding to any level at that particular instant, may be derived. Of course the movement must be smoothly made. This is the object of the height-adjusting screw on figure 4.

Alternatively a shadowgraph of the whole aspect of the tank may be made,

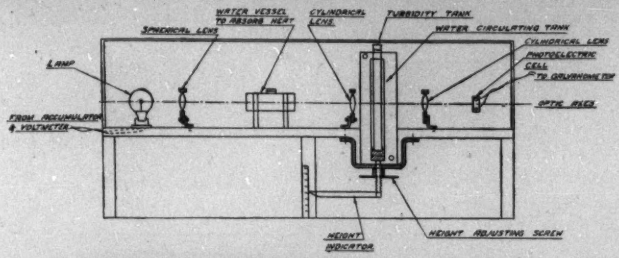


Figure 4. Apparatus for particle-size frequency.

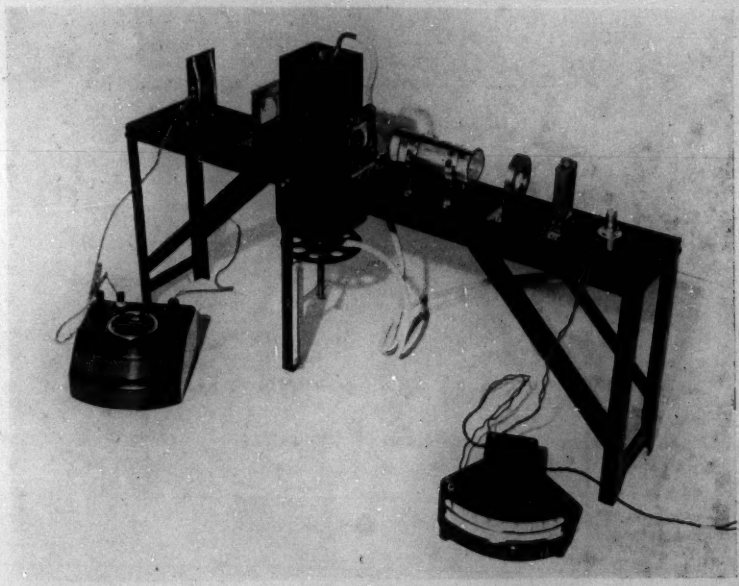


Figure 4 *a*. Photograph of particle-size apparatus.

using a wide-aperture lens to give a parallel beam embracing the whole tank and falling on a photographic plate of the same length as the tank behind it. On development, the plate will be graded for blackness in the same way as the opacity of the tank at corresponding levels. The plate is then passed through the narrow beam of light in place of the tank itself and readings of photo-electric current taken at the operator's leisure.

Such a shadowgraph is shown on figure 5 (below), with matching tints (above) and typical constant-time curves of opacity on figure 6.

It is, of course, possible to combine methods A and B by carrying the tank slowly down past the sampling light beam, e.g. by turning the height-adjusting screw of figure 4 at a constant rate by means of a motor and reduction gearing. When a specimen is known to carry a large range of particle size it is desirable,

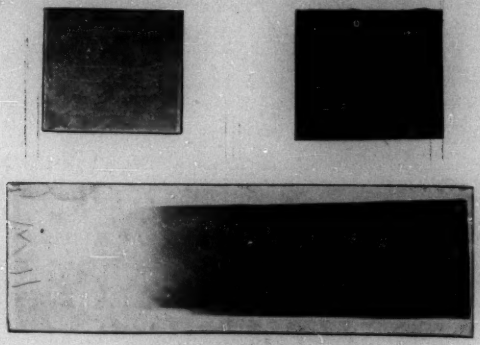


Figure 5. Typical shadowgraph during sedimentation.

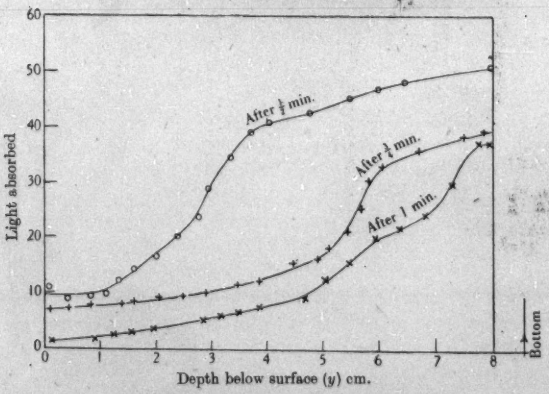


Figure 6. Analyses by constant-time method.

in order to record the large ones accurately, to use a rather viscous liquid, so that they do not fall too quickly and give rise to turbulence which may endanger the application of Stokes' law. With the constant-depth or constant-time method this may mean an inordinately long wait to "catch" the fine particles. If, however, the whole tank is lowered slowly so that the beam samples in succession the whole depth of liquid, save a few millimetres at top and bottom, the analysis may be expedited.

In this way the author has analysed a specimen of clayey soil from 120 μ down to 5 μ in a 4-inch tank containing a mixture of glycerine and water in 15 min. The distortion of the summation curve due to the changing depth of sampling with time (t) is readily corrected on the time axis in plotting. Thus, if H is the whole depth traversed from start to finish in time T, the sedimentation equation becomes

$$d = \frac{\sqrt{H}}{C} \cdot \sqrt{\left(\frac{1}{t} - \frac{1}{T}\right)}, \qquad \text{since} \quad \frac{H-h}{t} = \frac{H}{T}.$$

Another question which has been raised is that of the application of turbidity methods to estimate the specific surface and, if possible, the size-frequency curve for a suspension of liquid drops in another liquid medium, as in an emulsion. Provided the two liquids have sufficiently different refractivities, there seems no reason why the method described here should not be used. Bond found that droplets of density ρ fell in a lighter liquid of density σ in accordance with Stokes' law, provided they did not exceed a critical radius equal to $\sqrt{\frac{\tau}{(\rho-\sigma)}}$, where τ is the interfacial tension. The concentration of the disperse phase must not be so great that droplets can coalesce by collision before they reach the base of the sedimentation tank.

It would be interesting to apply this technique to a sample of fog collected in the open air, and allowed to fall in a vertical cylinder initially open to the atmosphere, past the sampling light beam. At the commencement of the analysis the top and bottom of the cylinder would be closed by shutters. The operation of these shutters without setting up currents in the portion of the atmosphere shut off would be the major experimental difficulty.

C. *Centrifuge method*

When, due to the fineness of the average particle, a suspension would take an inordinate time to settle under gravity, centrifuging may be resorted to to hasten the process. After this has continued for a sufficient time the shadow method (B) is adopted *in situ*. In the writer's apparatus (figure 7) a small

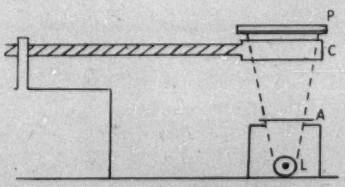


Figure 7. Centrifuge for shadowgraphs.

tank C (6 × 4 × 2·5 cm.) with quartz sides is fitted into a slot in a wooden disc which, mounted on the vertical axis of an electric motor, forms the centrifuge, shown in elevation. The source of light is the mercury arc L and ultra-violet-pass filter A. With the suspension in place the full voltage is turned on to the motor, and after a lapse of time a shutter is opened above the mercury lamp so that

an exposure of the photographic plate, P, is made. The shadowgraphs have the usual features of figure 5 and are analysed in the same way.

This table shows an analysis of specimens of titanium oxide in acetone. The figures in brackets are the corrected values, obtained by dividing the actual figures for each size by the ordinates of the curve of figure 1.

Mean diameter (μ)	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8
Specimen 1	5(2)	16(7)	28(16)	20(14)	16(15)	8	5	2
Specimen 2	14(5)	14(6)	22(13)	18(13)	14(13)	9	6	3

§ 5. CONVECTION IN TURBIDIMETERS

It has been suggested that results in the turbidimeter may be affected by convection, set up by the concentrated light beam. It should be noted that this is already guarded against by (1) water circulation round the tank, (2) absorption of heat rays before striking the tank. However, an experiment has been made to check this.

The turbidimeter has been set up, substantially as shown in figure 4 (except for the water circulation round the tank), in a cool ventilated room.

The test was carried out as an experiment in diffusion. The tank was filled with distilled water, and a layer of concentrated potassium permanganate carefully let into the bottom from a pipette.

This gradually diffused upwards and coloured the clear water above, eventually causing a reduction in the photoelectric current produced by the narrow beam of light (1 inch from bottom of tank). The experiment was allowed to proceed for seven hours and readings taken every hour or so, firstly with the beam of light shining on the cell continuously for the seven hours, secondly with the light turned on only at the instants of taking readings. There was no difference which could be ascribed to convection in the cell ; further, the diffusion coefficient from these results works out at its recognized value.

There is another type of convection which has previously been observed when a cluster of large particles falls rapidly through a liquid. Currents are sometimes produced in the fluid which they drag with them. When, therefore, a suspension contains large heavy particles ($>100\,\mu$), it is recommended to slow down the sedimentation by increasing the viscosity of the fluid.

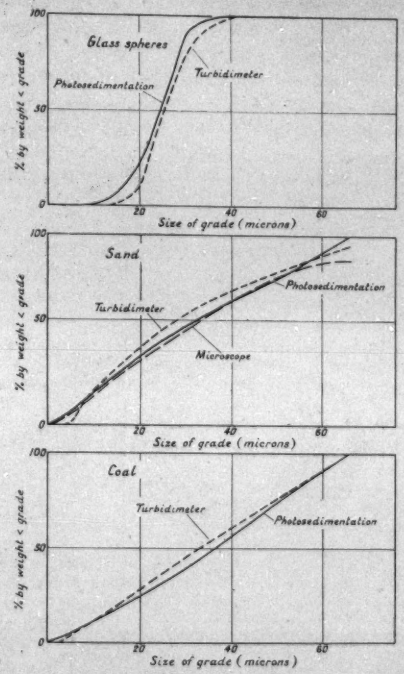
§ 6. MEASUREMENT OF SPECIFIC SURFACE

The measurement of the specific surface (= total area of 1 gm. of powder) in a turbidimeter is possible if one takes readings of the photoelectric current with a given mass in suspension (1) with the powder maintained in suspension by stirring, (2) with the liquid alone. It can also be got by integrating the "summation curve". This measurement is only feasible if *all the particles obey the same light-extinction law*, which means, in practice, that particles below $2\,\mu$ must be removed by decantation or by centrifuging.

D. *General notes on the methods*

Apart from counting individuals under the microscope, the sedimentation method is the only one in current use for mechanical analysis. When rough

estimates only are required, samples at constant depth may be abstracted from the suspension from time to time and literally weighed. The objections to this procedure are (1) that it disturbs the remaining suspension, (2) that it requires large quantities and high concentration to get enough in each sample for accurate weighing. Sometimes a hydrometer is floated in the liquid and readings of the effective density taken from time to time. This is free from the first objection, but the interpretation of the readings is uncertain. Yet again the sediment may be allowed to fall on a balance pan near the bottom of the tank. The increase



Figures 8, 9, 10. Comparison of photo-sedimentation and turbidimeter analyses.

in weight on the pan should give the summation curve, but in fact errors arise due to changes in effective density of the suspension below the pan, altering its buoyancy as the solid is cleared from below while it settles.

Probably the most accurate method is to take a photograph with time exposure of the paths of individual particles during sedimentation and compute their respective velocities from the traces on the negative (Carey and Stairmand, 1938). However, this is tedious, since many exposures must be made before a sufficient statistical sample can be obtained to give the distribution curve. Data correlating the photographic ("photo-sedimentation") with the photoelectric method are given in figures 8, 9, 10.

In all sedimentation work the choice of a method of dispersion is important if consistent results on a specimen are to be obtained. At the same time the dispersion must not be so vigorous that the particles are abraded and so do not represent the specimen in its natural state. In water suspensions the addition of a little sodium silicate, ethylene glycol or common glue will facilitate dispersion. More work needs to be done on this point.

Further work is also needed to elucidate the laws of sedimentation and of light extinction by particles which depart markedly from the spherical form. In this case only the *effective mean radius* can be deduced from the velocity of sedimentation.

Svedberg and Rinde (1928) remark that the particle must possess a rather high degree of asymmetry if this is to make a marked difference. They give the following table to show the ratio between the radius R of an ellipsoidal particle and that, r, of a sphere of equal mass. The lengths of the axes are a and c, of which the latter is greater. The suffixes a or c for R indicate that the equivalent radius is deduced on the supposition that the ellipsoids settle with this particular axis vertical.

Ratio of axis of ellipsoid	Disc (c, c, a)		Rod (c, a, a)	
a/c	R_a/r	R_c/r	R_a/r	R_c/r
0·1	0·870	0·783	0·900	0·751
0·2	0·942	0·824	0·978	0·849
0·3	0·974	0·876	1·008	0·902
0·4	0·992	0·911	1·020	0·934
0·5	1·000	0·937	1·024	0·956
0·6	1·004	0·956	1·022	0·971
0·7	1·006	0·972	1·018	0·982
0·8	1·005	0·983	1·013	0·990
0·9	1·003	0·992	1·009	0·995

§ 7. ACKNOWLEDGEMENTS

The author is pleased to make acknowledgement of a grant from Messrs. Imperial Chemical Industries, Ltd., which enabled the latter stages of this investigation to be carried out, and also for permission to publish the results; also to Mr. W. F. Carey and to Miss E. M. Halton, of the Engineering Department of that organization, for their helpful co-operation.

Note added in proof

Since this account was written, a paper by Schweyer (1942) has appeared, adversely criticizing the photoelectric method of determining particle-size frequency according to the technique of Wagner. The latter developed independently of the writer a method which, though the same in principle, is different in calibration (Wagner, 1933). He makes the arbitrary assumption that the mean size of all particles below 7·5 μ is 3·75 μ. Schweyer shows that though this may be true for certain cements, an assumption of a somewhat

different value for this mean makes a big difference to the deduced (size, frequency) curve. The present writer suggests that this discrepancy is removed by the method of calibration described in this lecture.

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DISCUSSION

Mr. G. S. FAWCETT. Dr. Richardson has kindly referred to a paper published by J. Hewitt and myself on *The Measurement of Turbidity and Fluorescence using the Lovibond Tintometer fitted with the Rothamsted Device*, and I am glad to hear him confirm our findings that in the lower ranges of turbidity (up to 100 parts per 100,000 using a fuller's-earth suspension) the best results are obtained by reflected light, above which there is a distinct flattening of the curve, and results using transmitted light give much better discrimination.

The method employed by Dr. Richardson of measuring transmitted light by means of a photoelectric cell is of considerable interest and a very useful contribution to the subject.

I shall be interested to hear what size of cell Dr. Richardson used in these experiments and to hear what influences and decides the size of the cell employed. The question is not asked without a practical object in view, because my firm (The Tintometer, Ltd.) are receiving frequent requests to supply larger and larger fused or sintered cells, and with the equipment available 8 in. in height is the maximum we have so far been successful in making to our own satisfaction. If larger cells are ultimately found necessary, it may be expedient to resort to using wide-bore tubes with optical faces inserted in the sides of the tubes.

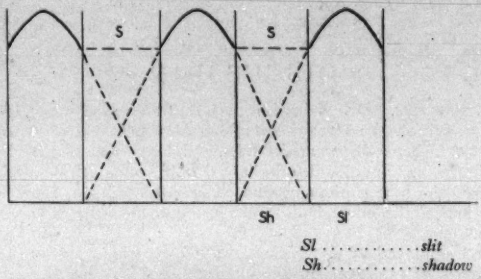
Mr. E. R. DAVIES. I should like first to record my appreciation of a paper on a subject of great technological importance and one which has not received the attention it deserves. Can Dr. Richardson's general equation $I_x = I_0 e^{-kxc}$ really be considered to hold generally? It assumes a simple blocking effect; and is it not true that it excludes the possibility that any light scattered by the particles may enter the measuring apparatus ?

Mr. J. W. PERRY. Dr. Richardson refers to the screening of one particle by another being eliminated by the restriction to "low" concentrations, these being presumably so low that $I_{xc} = I_0 e^{-kxc}$ becomes $I_0 - I_{xc} = kxcI_0$, to cause the *shadow effect* mentioned. Under widely varying conditions—with varying x specifically—would Dr. Richardson adjust the concentration inversely to maintain this restriction, and can he say if recalibration would then be necessary ?

From the more general point of view, turbidity is often regarded as an optical property, in which case e^{-kxc} represents a complex relation, which may be further complicated by the presence of absorption. But turbidity may also, in some connections, denote a mechanical property of a suspension. Both meanings are, moreover, sanctioned by

English usage (" turbid " = " thick *or* opaque with suspended matter ".—*O.E.D.*). I should here like to suggest that the time is ripe for a clarification of this position physically and a consideration of an acceptable, tentative, basic scale of optical turbidity and of standards, suitable for use over a range avoiding extreme conditions and the necessity for problematic corrections.

Dr. G. Spiegler. Turbidity plays an important part within a photographic emulsion. Lack of sharpness on a photographic negative caused by the film or plate is due to the stray light scattered at the grains from the " lights " into the " shadows ". With a close sequence of lights and shadows, the shadows, fully transparent with ideal definition, become less transparent with decreasing definition because of the light being scattered sideways. Whilst the direct cause of any lack of sharpness is a gradual falling-off of the intensity of light across the boundary from full light to the value in the shadow, this transition curve becomes represented as a sequence of tone values on the picture through the characteristic curve of the emulsion. Whilst low exposure does not bring out the weak intensities, increased exposure converts them into blackenings of increasing density. With the drop of transparency within the shadows due to prolonged exposure the visibility of details, blackened out in this way, decreases. The simplified sketch here shown (in



Long exposure blackens the shadows, *S*;
short exposure leaves the slits distinct, full lines.

which a close sequence of narrow slits—say of 1 mm. width—is supposed to form the object) shows this clogging of the shadows. Thus, increased exposure diminishes the resolving power. Conversely ideal definition would imply invariance of the transparency of the shadows with the exposure.

With the aid of a method of measuring the definition of intensifying screens developed by two former co-workers of mine (Juris and Rudinger *), I have measured the resolving power of films. The picture of a grid with holes of 0·5 mm. diameter, spaced at 0·65 mm. apart, was taken. With sufficient exposure to blacken the holes so that the transparency of the image is chiefly due to the shadows, the photoelectric current passing, if the picture of the grid is laid upon a light cell, is due to the light transmitted by the sum of the shadows, whilst the holes contribute almost nothing. Any decrease of this " shadow current " can be converted into the ratio of the intensity of the stray light scattered into the shadows to the intensity of the full light, by means of a graded optical wedge. An x-ray film, illuminated from one side only, was found to scatter 4 %.

A critical value to which the exposure can be extended without the details clogging up can be assigned to any photographic material.

I should like to acknowledge the helpful support which Professor Mayneord has given these investigations.

* Juris, K. and Rudinger, G., 1937. " Ein objektives Verfahren zur Prüfung der Zeichenschärfe von Verstärkungsfolien ohne Mikrophotometer und ein Schärfemessapparat ". *Fortschr. Röntgenstr.* **56**, 548.

AUTHOR's reply. In reply to Mr. Fawcett, I think a cell 8 in. high should be adequate for turbidity work. I am afraid I have no data to "confirm" Messrs. Fawcett and Hewitt's work, except, indirectly, that some measurements which were made for me of the turbidity of certain natural water samples in their apparatus fell into line with other evidence which I had.

I do not agree that the general equation quoted by Mr. Davies does not cover scattering. It is perfectly general except in so far as large concentrations may invalidate the assumption of an absorption coefficient proportional to the concentration. In the general case, of which Rayleigh's equation for particles smaller than the wave-length and the one I use for particles considerably larger than the wave-length are particular instances, k is a function of the particle diameter and of the wave-length.

As regards the point raised by Mr. Perry, experiments in which the concentration of a soil suspension varied between 0·2 and 2 gm. per litre gave the same distribution curve (within operational tolerance) below 0·7 gm. per litre, but progressive over-emphasis of the finer particles at higher concentrations. In practice one uses about 0·5 gm. per litre.

I agree with him that a discussion of definitions and units of turbidity is desirable.

THE DETERMINATION OF THE COEFFICIENT OF KINETIC FRICTION

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Newport, Mon.

MS. received 29 *July* 1942; *in revised form* 22 *August* 1942

ABSTRACT. Movement is caused by applying a succession of uniformly increased forces. The accelerations produced are in turn plotted, by means of the Vector Trolley Apparatus, against a constant acceleration at right angles, and by direct reading the *resisting force* is measured. The surfaces in contact remain constant. The reactions between the surfaces are increased by adding loads to the moving body.

WHEN one of two bodies in contact is made to slide upon the surface of the other, it is generally necessary to apply a force in excess of some minimum value before there is any movement. As one surface rubs over the other, a force acts to oppose the motion. When this resistance breaks down and motion takes place, the body moves forward with an acceleration, showing that the force being applied is in excess of the "Resisting Force". In this present investigation it is this resisting force which is determined.

By the application of the principle of the *parallelogram of accelerations* it is possible to compare the different accelerations by compounding each successive acceleration (represented in figure 1 by OA_1, OA_2, OA_3 ....) with the same constant acceleration in a direction at right angles to that of those being compared. By so doing a series of resultant accelerations is obtained, as shown by OP_1, OP_2, OP_3 ....

This can now be done practically by the *Vector Trolley Apparatus*,* briefly noted and illustrated in *Nature* on 25 February 1925, described, with many applications, in the *Journal of Scientific Instruments* in July and August of that year, and demonstrated at the Annual Exhibition of the Physical and Optical Societies in January 1930.

* Makers: Messrs. G. Cussons, Ltd., Lower Broughton, Manchester, 7.

It will be recalled that the Vector Trolley consists of a wide square-framed trolley capable of motion along a track and carrying upon its back, as it were, a small trolley capable of motion along a track which can be arranged normal or inclined to the path of the main trolley, and with a motion independent of that of the main trolley. The trolleys are actuated by adjustable falling weights, and the combined effects of the independent motions of these trolleys in directions inclined at suitable angles to one another are recorded by means of a tracer.

The two trolleys are released simultaneously; thus, with the hanging masses falling freely, the smaller trolley carrying the marking brush will move with a motion which is a combination of its own motion with that of the main trolley, the combination of two accelerations. If variations to the force applied to the main trolley be made by transferring masses from the frame to the hanging masses (thus keeping the total mass moved constant), and combining the resulting accelerations successively with a constant acceleration of the cross-trolley, it will be seen that the accelerations of the frame trolley, as represented by YP_1, YP_2, YP_3, will be proportional to the applied force. If there is an opposition, or a force resisting the motion, then the magnitude of the force causing the motion

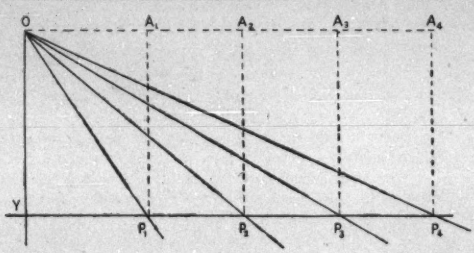

Figure 1.

will be the excess of the applied force over the opposing force, and the acceleration will be proportional to this difference.

The trolleys are free-moving, all friction being compensated by means of the very light weight-hangers. The sliding body to be considered is attached rigidly by means of a rod to the main trolley. As this trolley moves along its track, this body is dragged behind, so that the force which actuates the main trolley actuates the whole system. The action of the sliding body is to oppose the motion of the system. When motion ensues, the magnitude of the force causing the motion is the excess of the applied force over the opposition, and, as stated in the last paragraph, the resulting acceleration is proportional to this difference.

The cross-trolley is set up to move with an acceleration in a direction at right angles to the direction of the main trolley, as seen in the accompanying photograph (figure 2). When the whole is released, a trace of OP_1 is obtained. If the applied force is now increased by unity by transferring unit mass from the frame to the weight-hanger, upon release a second trace OP_2 is obtained. Further traces as OP_3, OP_4, etc., are obtained by successively transferring other unit masses.

If a line Y_1P be drawn through a point Y_1 on the zero direction OY of the cross-trolley parallel to the direction of the main trolley, then the intercepts on this line between successive traces (i.e. P_1P_2, P_2P_3, etc.) will be observed to be equal and proportional to the acceleration which would result from the application of unit force to the whole if there were no opposition. By dividing the length of the intercept (YP_1) between the zero direction of the cross-trolley and the first trace by the length of one of these equal intercepts (P_1P_2) we have the magnitude of the effective force actuating the system during the first trace. The applied force is noted, so that by subtraction we have the magnitude of the friction opposing the sliding of the body along the plane while the body was actually in motion. If the line YP is drawn such that the lengths P_1P_2, P_2P_3, etc., are

Figure 2.

measured in convenient units, as inches or centimetres, then by placing a ruler suitably along the line the magnitude of the force of friction can be read directly from the ruler. For example, if the force applied to give OP_1 was 0·5 lb. and to give OP_2 was 0·6 lb., then by placing the ruler along YP, with the 5-inch mark at P_1 and the 6-inch mark at P_2, then the reading on the ruler at Y, 3·7, will give directly the value of the friction (or resisting) force as 0·37 lb.

Since the whole motion takes place upon a horizontal plane, the vertical reaction between the sliding body and the plane will be equal in magnitude to the gravitational attraction upon the body.

It will be seen from the photograph (figure 2) that the sliding body carries upon its back a tray to accommodate masses which may in effect increase the reaction between the surfaces without increasing the areas in contact.

The following results were recently obtained by this method (see page 66):—

Coefficients of kinetic friction

Sliding surface (area 7″×5″)	Reaction or load R (lb. wt.)	Opposing or friction force F (lb. wt.)	F/R or μ
A. *Fixed surface—Mahogany*			
White wood	1·0	0·31	0·31
	1·5	0·47	0·31
	2·0	0·61	0·305
	2·5	0·763	0·305
Oak	1·15	0·31	0·27
	1·5	0·41	0·273
	2·0	0·535	0·268
	2·5	0·652	0·268
Leather	1·15	0·69	0·63
	1·5	1·0	0·66
	2·0	1·3	0·65
	2·5	1·6	0·64
	3·0	1·8	0·60
	3·5	2·1	0·60
Steel	1·5	0·535	0·357
	2·0	0·715	0·357
	2·5	0·89	0·356
Rubber	1·4	0·745	0·54
	2·0	1·11	0·55
B. *Fixed surface—Steel*			
Steel	1·5	0·68	0·45
	2·0	0·88	0·44
	2·5	1·1	0·44
"Ferodo"	1·47	0·7	0·476
	2·1	0·99	0·47
	2·5	1·2	0·48
Rubber	1·4	0·94	0·67
	2·0	1·344	0·67
C. *Fixed surface—Mastic asphalte.* (*Smooth-surface specimen of a road surface*)			
Rubber	1·4	1·055	0·753
	2·0	1·51	0·755
D. *Fixed surface—Concrete.* (*Specimen of a road surface*)			
Rubber	1·4	0·86	0·61
	2·0	1·215	0·605

THE SIGNIFICANCE OF COUPLING BETWEEN SHEAR AND EXTENSION IN THE ELASTIC BEHAVIOUR OF WOOD AND PLYWOOD

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Communicated by W. W. Barkas, 28 August 1942

ABSTRACT. The effect on the elastic behaviour of wood and plywood of coefficients such as s'_{16} (coupling coefficients) in the generalized Hooke's Law is considered. It is shown that for specimens cut at angles to the principal directions, these coefficients may differ from zero. This implies a difference between " pure " and " free " elastic constants for such specimens, a difference which is verified experimentally. The existence of coupling coefficients introduces uncertainties into the calculation of the free Young's and rigidity moduli from the frequencies of flexural and torsional vibrations, but it is shown that the free constants can be measured to sufficient accuracy by working on specimens of suitable length. Finally, it is shown that the elastic behaviour of plywood cut at an angle to the grain may be determined as much by the pure elastic constants of the individual plies as by the free constants.

PART I. THEORETICAL

§1. INTRODUCTION

The generalized Hooke's Law (Love, 1927, pp. 97, 99, 106) relates all possible components of stress to all possible components of strain. Thus the extensional strain in the x direction (x_x) is given by

$$x_x = s_{11}X_X + s_{12}Y_Y + s_{13}Z_Z + s_{14}Y_Z + s_{15}Z_X + s_{16}X_Y,$$

while the shear strain in the xy plane (x_y) is given by

$$x_y = s_{61}X_X + s_{62}Y_Y + s_{63}Z_Z + s_{64}Y_Z + s_{65}Z_X + s_{66}X_Y, \text{ etc.}$$

In these equations X_X, Y_Y and Z_Z are extensional stresses, Y_Z, Z_X and X_Y are shear stresses, while the coefficients s_{ik} are constants of the material. In the most general case, these coefficients are subject only to the restrictions $s_{ik} = s_{ki}$, but there may be other relations among them, depending on the symmetry group to which the material belongs (Love, 1927, p. 154; Voigt, 1928, p. 588).

Some of the coefficients are easily interpreted in terms of familiar elastic constants. Thus, if

E_X = Young's modulus in the x direction,

$\sigma_{XY} = \dfrac{\text{contraction in } Y \text{ direction}}{\text{extension in } X \text{ direction}}$ for extensional stress in x direction, and

G_{XY} = rigidity modulus for shear stresses in the x and y directions,

then

$$s_{11} = 1/E_X; \quad s_{12} = -\sigma_{XY}/E_X; \quad s_{13} = -\sigma_{XZ}/E_X; \quad s_{66} = 1/G_{XY}, \text{ etc.}$$

The remaining coefficients, such as s_{16}, s_{46} (for which the name "coupling coefficients" is proposed), are not so easily interpreted, since they involve somewhat less familiar ideas. The former, in fact, relates a shear stress to an extensional strain and *vice versa*, while the latter relates a shear stress in one plane to a shear strain in a perpendicular plane. For an isotropic solid, the coupling coefficients are always zero, and they are, therefore, never encountered in elastic theory as applied to isotropic materials.

Wood can be classified as having approximately rhombic symmetry (Hörig, 1931), and its elastic behaviour can thus be specified in terms of 9 independent coefficients: s_{11}, s_{22}, s_{33}, s_{44}, s_{55}, s_{66}, s_{12}, s_{13}, s_{23}. These coefficients are measured with respect to the three principal directions, the longitudinal (L), the radial (R) and the tangential (T). Thus, provided attention is confined to the principal directions, the coupling coefficients are zero. Examination of Voigt's equations (1928, p. 593) for transforming the elastic coefficients to directions other than the principal ones shows that in these cases the coupling coefficients for wood may attain finite values. This fact has not previously been pointed out, and the purpose of the present paper is not only to emphasize it, but also to show that it may be of practical importance in some problems of the elastic constants of wood and in particular of plywood.

§ 2. GENERAL THEORY

Confining attention to the xy plane, the generalized Hooke's Law for a state of plane stress becomes

$$\begin{aligned} x_x &= s'_{11}X_X + s'_{12}Y_Y + s'_{16}X_Y, \\ y_y &= s'_{12}X_X + s'_{22}Y_Y + s'_{26}X_Y, \\ x_y &= s'_{16}X_X + s'_{26}Y_Y + s'_{66}X_Y, \end{aligned}$$

where the dashes are used to indicate that attention is not necessarily confined to the principal directions.

If the only stresses acting are X_X and X_Y, then

$$\begin{aligned} x_x &= s'_{11}X_X + s'_{16}X_Y, \\ x_y &= s'_{16}X_X + s'_{66}X_Y. \end{aligned}$$

By setting each of the stresses and strains in turn equal to zero, four cases arise:—

(A) $$x_x = 0 \text{ and } \frac{X_Y}{x_y} = G_P = 1\Big/\left(s'_{66} - \frac{s'^2_{16}}{s'_{11}}\right),$$

where G_P is defined as the "pure" rigidity modulus (Voigt, 1928, p. 638; Brown, 1940).

(B) $$x_y = 0 \text{ and } \frac{X_X}{x_x} = E_P = 1\Big/\left(s'_{11} - \frac{s'^2_{16}}{s'_{66}}\right),$$

where E_P is the pure Young's modulus.

(C) $$X_X = 0 \text{ and } X_Y/x_y = G_F = 1/s'_{66},$$

where G_F is the "free" rigidity modulus (Voigt, 1928, p. 638; Brown, 1940).

(D) $X_Y=0$ and $X_X/x_x=E_F=1/s'_{11}$,

where E_F is the free Young's modulus.

Thus, collecting results,

$$\frac{G_F}{G_P}=\frac{E_F}{E_P}=1-\frac{s'^2_{16}}{s'_{11}s'_{66}}=1-\epsilon. \quad \ldots\ldots(1)$$

These equations show that if the constant s'_{16} exists, then there is a difference between the pure and the free Young's moduli. The pure constant corresponds to the simultaneous application of shear and extensional stresses such that the

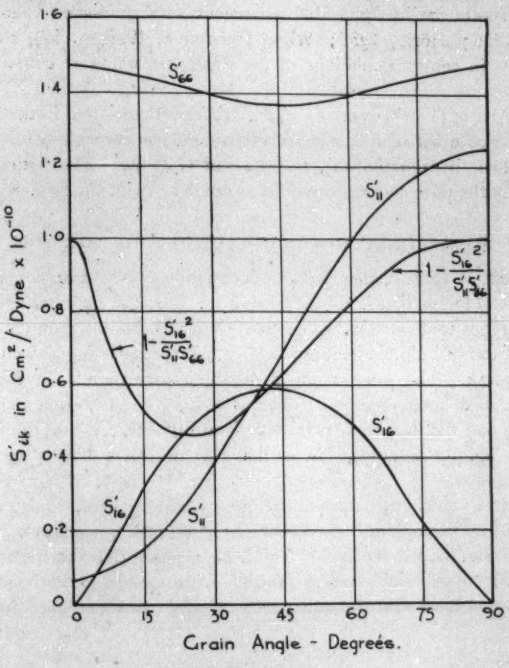

Figure 1.

strain is purely extensional. The free constant corresponds to the application of an extensional stress only, and the resulting strain is a combination of shear and extension. The argument in the case of rigidity modulus is the same, except that the rôles of shear and extension are interchanged.

For plane stress, only four of the nine principal elastic coefficients of wood are needed. In the LT plane these are:

$$s_{11}=1/E_{\mathrm{L}};\quad s_{22}=1/E_{\mathrm{T}};\quad s_{12}=-\sigma_{LT}/E_{\mathrm{L}};\quad s'_{66}=1/G_{\mathrm{LT}}.$$

Now consider a direction in the LT plane at an angle θ with the L direction.

Then Voigt's equations for transforming the elastic coefficients to the new orientation become:—

$$\left.\begin{aligned} s'_{11} &= s_{11}\cos^4\theta + (2s_{12}+s_{66})\sin^2\theta\cos^2\theta + s_{22}\sin^4\theta, \\ s'_{66} &= 4(s_{11}+s_{22}-2s_{12})\sin^2\theta\cos^2\theta + s_{66}(\cos^2\theta-\sin^2\theta)^2, \\ s'_{16} &= -\sin\theta\cos\theta[2s_{11}\cos^2\theta - 2s_{22}\sin^2\theta - (2s_{12}+s_{66})(\cos^2\theta-\sin^2\theta)]. \end{aligned}\right\} \quad \ldots\ldots(2)$$

Equations (2) are illustrated in figure 1 for a timber with the following assumed constants (unit $= 10^{10}$ dynes/cm^2):—

$$E_L = 15{\cdot}0; \quad E_T = 0{\cdot}8; \quad G_{LT} = 0{\cdot}675; \quad \sigma_{LT} = 0{\cdot}4.$$

These constants are in reasonable agreement with published values for birch (Stamer, 1935; Hörig, 1935; Wood Handbook, 1935, p. 57), and are used again below in some calculations of the elastic constants of commercial birch plywoods.

The values of $(1-\epsilon)$ (where $\epsilon = s'^2_{16}/s'_{11}s'_{66}$) are also plotted in the same figure. At grain angles other than 0° and 90° this quantity is necessarily less than unity, and, therefore, by equation (1), $E_P > E_F$ and $G_P > G_F$. The figure shows that theoretically the pure constants may be as much as twice the free constants.

§3. APPLICATION TO BENDING AND TORSION

If s'_{16} differs from 0, there will be interaction between torsion and flexure. Assuming the interaction is similar, the above ideas can be extended to include these two types of distortion. The free Young's modulus then corresponds to the application of a simple bending moment, so that twisting as well as bending can take place. The pure Young's modulus corresponds to the combined application of a bending moment and a twisting couple, such that torsion is suppressed and the distortion is purely flexural. Here, again, the argument is similar for rigidity modulus, but in this case the rôles of torsion and flexure are interchanged.

In order to obtain an approximate theory of the effect, consider a strip of wood cut at an angle to the grain, and having length l, breadth b and depth d ($l \gg b \gg d$). Let the x axis coincide with l, the y axis with b and the z axis with d. For a material in which s'_{16} is zero, if the bar is clamped at one end and deflected a distance z by a force P at the other, then

$$z = 4Pl^3/bd^3E. \quad \ldots\ldots(3)$$

If s'_{16} is not zero, it is assumed that (3) can still be used, provided E is replaced by the Young's modulus appropriate to the conditions of the experiment.

If, on the other hand, the bar is twisted about the x axis by a couple C, it is assumed that

$$\phi = lC/\mu bd^3G, \quad \ldots\ldots(4)$$

where ϕ is the angle of twist at the end of the strip and G is the rigidity modulus appropriate to the conditions of the experiment; μ is the St. Venant factor which tends to 1/3 as d/b tends to 0. The calculation of this factor in the case of wood has been dealt with in a previous paper (Hearmon and Barkas, 1941, subsequently referred to as (I)) and, among others, by Hörig (1936).

Since the wood is cut at an angle to the L direction, s'_{16} is not zero, and the strip will therefore both twist and bend under the strain of a simple end-force P or a simple couple C. Let

$$\left.\begin{aligned} P &= \alpha_{11}z + \alpha_{12}\phi, \\ C &= \alpha_{12}z + \alpha_{22}\phi; \end{aligned}\right\} \quad \ldots\ldots(5\,a)$$

$$\left.\begin{aligned} z &= \beta_{11}P + \beta_{12}C, \\ \phi &= \beta_{12}P + \beta_{22}C. \end{aligned}\right\} \quad \ldots\ldots(5\,b)$$

In accordance with (3) and (4) we can put

$$\alpha_{11} = bd^3E_P/4l^3; \quad \alpha_{22} = bd^3\mu G_P/l; \quad \beta_{11} = 4l^3/bd^3E_F; \quad \beta_{22} = l/G_Fbd^3\mu.$$

If equations (5 *a*) are solved to express z and ϕ in terms of P and C, and these coefficients equated to those of (5 *b*), we have

$$\beta_{11} = \frac{\alpha_{22}}{\alpha_{11}\alpha_{22} - \alpha_{12}^2}; \quad \beta_{12} = \frac{\alpha_{12}}{\alpha_{12}^2 - \alpha_{11}\alpha_{22}}; \quad \beta_{22} = \frac{\alpha_{11}}{\alpha_{11}\alpha_{22} - \alpha_{12}^2},$$

and, therefore, bearing equations (1) in mind,

$$\alpha_{12}^2 = \frac{\alpha_{22}(\beta_{11}\alpha_{11} - 1)}{\beta_{11}} = \frac{b^2d^6\mu s'^2_{16}}{4l^4(s'^2_{16} - s'_{11}s'_{16})^2}; \quad \beta_{12}^2 = \frac{4l^4s'^2_{16}}{b^2d^6\mu}.$$

It can be shown (Timoschenko, 1934, p. 244) that if $b \gg d$, and if the effect of the short sides of the cross-section on the stress distribution is neglected, then the shear stress in torsion is distributed over the cross-section in the same way as extensional stress in bending. The above treatment of the coupling between torsion and bending assumes this equality of distribution throughout. Thus the treatment, and any results deduced from it, can be approximate only.

§ 4. FREQUENCIES OF FLEXURAL AND TORSIONAL VIBRATIONS

In paper (I), results were given of determinations of E and G for wooden strips cut at an angle to the grain. The methods were described in (I), but they depended on the frequencies of torsional and flexural vibrations. It was pointed out that some of the results were possibly in error due to coupling between torsional and flexural vibrations, and it is therefore of interest to examine this possibility more closely.

Suppose a body of mass M is rigidly attached to the free end of the strip, and let its moment of inertia for rotation about the x axis be I. Then the equations of motion are

$$\left.\begin{aligned} \frac{d^2z}{dt^2} + \frac{\alpha_{11}}{M}z + \frac{\alpha_{12}}{M}\phi &= 0, \\ \frac{d^2\phi}{dt^2} + \frac{\alpha_{12}}{I}z + \frac{\alpha_{22}}{I}\phi &= 0. \end{aligned}\right\} \quad \ldots\ldots(6)$$

Solving the first of these for ϕ and substituting in the second, we find

$$\frac{d^4z}{dt^4} + \left(\frac{I\alpha_{11} + M\alpha_{22}}{IM}\right)\frac{d^2z}{dt^2} + \left(\frac{\alpha_{11}\alpha_{22} - \alpha_{12}^2}{IM}\right)z = 0.$$

Assuming, in the usual way, $z = e^{kt}$ and ignoring irrelevant solutions, we obtain

$$z = A \sin(2\pi\nu_1 t + \psi_1) + B \sin(2\pi\nu_2 t + \psi_2),$$

where ν_1 and ν_2 are the frequencies.

A, B, ψ_1 and ψ_2 are arbitrary constants governed by the boundary conditions.

Putting for brevity $\Delta = \alpha_{12}^2/(I\alpha_{11} - M\alpha_{22})$, then approximately

$$\nu_1 = \frac{1}{2\pi}\sqrt{\left(\frac{\alpha_{11}}{M} + \Delta\right)}; \qquad \ldots\ldots(7)$$

$$\nu_2 = \frac{1}{2\pi}\sqrt{\left(\frac{\alpha_{22}}{I} - \Delta\right)}. \qquad \ldots\ldots(8)$$

These expressions give the perturbation of the frequencies due to coupling. If $s'_{16} = 0$, $\alpha_{12} = \Delta = 0$, and we obtain two independent frequencies:

$$\frac{1}{2\pi}\sqrt{(\alpha_{11}/M)} \quad \text{and} \quad \frac{1}{2\pi}\sqrt{(\alpha_{22}/I)},$$

as can otherwise be seen from (6).

§ 5. EFFECT OF COUPLING IN YOUNG'S-MODULUS AND RIGIDITY-MODULUS MEASUREMENTS

(a) *Young's modulus*

Squaring (7) and inserting the values of Δ, α_{11}, α_{22} and α_{12}, we get

$$\frac{16\pi^2\nu_1^2 M_E l^3}{bd^3} = E = \frac{E_F}{1-\epsilon}\left[1 + \frac{4l^2 M_E \mu}{(I\,s'_{66} - 4l^2 M_E \mu s'_{11})} \cdot \frac{s'^2_{16}}{s'_{66}}\right], \qquad \ldots\ldots(9)$$

where E is the Young's modulus calculated directly from experimental results, no account being taken of s'_{16}.* If in (9) $I_E s'_{66}$ is negligible in comparison with $4l^2 M_E \mu s'_{11}$, then $E = E_F$. In other cases, E must be corrected by (9) to obtain E_F, which is the value finally required.

(b) *Rigidity modulus*

Proceeding similarly with (8), we obtain

$$\frac{4\pi^2\nu_2^2 I_G l}{bd^3\mu} = G = \frac{G_F}{1-\epsilon}\left[1 - \frac{I_G}{(I_G s'_{66} - 4l^2\mu M_G s'_{11})} \cdot \frac{s'^2_{16}}{s'_{11}}\right], \qquad \ldots\ldots(10)$$

where G is the rigidity modulus calculated directly from the experimental results, no account being taken of the existence of s'_{16}. In this case, if $4l^2\mu M_G s'_{11}$ is negligible in comparison with $I_G s'_{66}$, then $G = G_F$. Otherwise G must be corrected by (10) in order to obtain G_F.

In practice, the most convenient way of dealing with these errors is to minimize them by suitable adjustment of l. Owing to the large number of variables in (9) and (10), a complete discussion of the problem is not possible here. Figures 2 A and 2 B, however, show the variation of E/E_F and G/G_F with l for a particular case, corresponding approximately to the experiments described

* If the mass of the specimen and rotatory inertia effects in the zx plane are negligible, then $M = M_E$. Otherwise M_E is given approximately by the bracketed expression in equation (4) of paper (I).

in (I) and calculated from equations (9) and (10). It will be seen that by making l sufficiently long E_F can be measured directly to any desired accuracy, and by making l sufficiently short the same is true of G_F. Thus, in the particular case illustrated in figure 2, any length greater than about 30 cm. for E_F, or shorter than about 10 cm. for G_F, will give these two constants directly with an error of less than 2 %.

Although the reason underlying the fact was not known when (I) was written, it was found empirically that the best results for strips subject to these errors

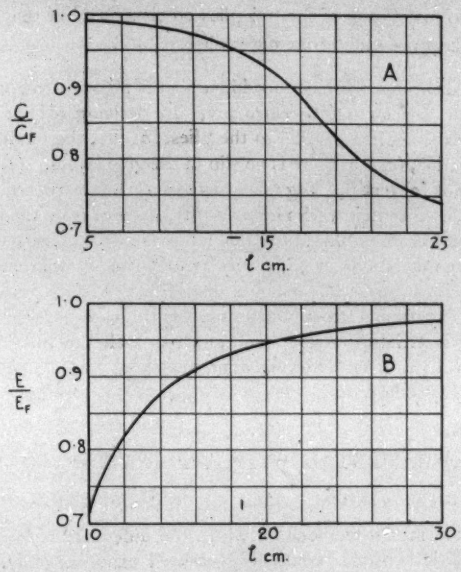

Figure 2.

were obtained when short lengths were used for G and long lengths for E. As a result, the corrections to be applied are not large, and do not exceed about 5 %. Since other errors may be as large as, if not larger than, this, detailed corrections are not given here. In §8 some new results are presented, which show that even when the correction is made as small as possible, the agreement is not appreciably improved, and it therefore seems justifiable to allow the earlier results to stand as originally given.

§6. THE ELASTIC CONSTANTS OF A LAMINATED MATERIAL

In dealing with plywood, the apparent Young's modulus and rigidity modulus of the material as a whole will be required in terms of the Young's moduli and rigidity moduli respectively of the individual layers, together with the thicknesses of the latter. The equations used later are

$$E_c t_n = E_1 t_1 + E_2(t_2 - t_1) + \ldots + E_n(t_n - t_{n-1}), \qquad \ldots\ldots(11)$$

$$E_b t_n^3 = E_1 t_1^3 + E_2(t_2^3 - t_1^3) + \ldots + E_n(t_n^3 - t_{n-1}^3), \quad \ldots\ldots(12)$$

$$G_t t_n^3 = G_1 t_1^3 + G_2(t_2^3 - t_1^3) + \ldots + G_n(t_n^3 - t_{n-1}^3), \quad \ldots\ldots(13)$$

where E_c is the apparent Young's modulus in compression-tension, E_b that in bending, and G_t is the apparent rigidity modulus in torsion.

$E_1, G_1, E_2, G_2 \ldots E_n, G_n$ are Young's and rigidity moduli of the respective plies, counted from the centre,

t_1 = half thickness of centre ply,

$t_2 - t_1$ = thickness of each of the two plies on either side of the centre, and

$t_n - t_{n-1}$ = thickness of each of the outer plies.

The material is supposed to have an odd number of plies and to be symmetrical in structure with respect to the centre ply. In deriving (11) it is assumed (*a*) that the stress is applied parallel to the plies, (*b*) that the strain is uniform over the cross-section, (*c*) that there is no slip in the glue layers. (12) is derived on the assumptions (*a*) that bending takes place in a plane perpendicular to that of the plies, (*b*) that the strain varies linearly with distance from the neutral axis, and (*c*) that there is again no slip in the glue layers (see, e.g., Hearmon, 1939).

In deriving the equations in §3, it was assumed as an approximation that where $b \gg d$, the shear stress in torsion is distributed over the cross-section in the same way as extensional stress in bending. If the same assumption is made for a laminated material in torsion about an axis parallel to the plies, then an approximation to G_t is given by an equation analogous to (12), and this is the basis on which (13) is derived.

PART II. EXPERIMENTAL, WITH APPLICATION TO PLYWOOD

§7. EXPERIMENTS WITH SIMPLE AND COMPOSITE BIRCH STRIPS

There are at least two methods by which the difference between the pure and the free elastic constants could be measured experimentally. The first would be to use an apparatus capable of either measuring or controlling each of the four variables, P, C, z and ϕ in equations (5). The second method is less direct, but in it the experimental conditions approximate closely to those obtaining in plywood. Since one of the main objects of the present investigation is an interpretation of the elastic constants of plywood, the second method was the one adopted.

The basis of this method is the fact that if the sign of the angle in equations (2) is changed, then the sign of s'_{16} is also changed. Examination of equations (2) shows that s'_{11} and s'_{66} are even functions of $\sin\theta$ and $\cos\theta$, so that $s'_{11}(\theta) = s'_{11}(-\theta)$; $s'_{66}(\theta) = s'_{66}(-\theta)$. On the other hand, s'_{16} is an odd function of $\sin\theta$ and $\cos\theta$ and $s'_{16}(\theta) = -s'_{16}(-\theta)$.

If, therefore, we take two strips, in one of which the grain angle is θ and in the other $-\theta$, and glue them together, the elastic constants of the composite strip should be the pure constants, whereas the constants of the individual strips should be the free constants. In the composite strip, in addition to the bending moment, the couples required to produce pure bending are supplied by the

interaction between the two strips, and the same is true of the bending moments required to produce pure torsion under the action of a twisting couple.

For purposes of discussion in the case of plywood, consider the torsion of a strip of 3-ply cut so that the grain angle in the outer plies is θ, and suppose the axis of the twisting couple is parallel to the length of the strip. Owing to the existence of s'_{16}, all the plies have a tendency to bend as well as to twist. Figure 1 shows that in a given quadrant, s'_{16} has the same sign throughout. In a normal plywood, the alternate plies are at right angles, and the grain angle of the inner ply is therefore $-(90-\theta)$. Thus the sign of s'_{16} for the inner ply will be opposite to that of s'_{16} for the outer plies, and the twisting of the plywood as a whole will be neither completely pure nor completely free, but intermediate in character between the two. In order that the twisting might be completely pure, a particular relation would have to exist between the thicknesses of the inner and outer plies and the two values of s'_{16} involved. Whatever the exact relation, it would only be satisfied by chance in a plywood as ordinarily made up, and further, even if satisfied at one grain angle, it would not be satisfied at any other. This fact alone makes it impracticable at the moment to do more than show that the pure constants of the constituent laminae may give a better approximation to the constants of the plywood than the free constants.

The experimental material in its original state consisted of sheets 60 cm. × 30 cm., of rotary-cut Canadian birch veneer, 0·16 cm. thick. A number of strips 30 cm. × 2·5 cm. were cut from these sheets in such a way that the lengths of the respective strips lay at the following angles with the grain of the veneer:—

$$0°,\ 15°,\ 22{\cdot}5°,\ 30°,\ 45°,\ 60°,\ 67{\cdot}5°,\ 75°,\ 90°.$$

Groups containing either five or seven strips were cut at each grain angle, then conditioned to 60 % relative humidity, and the values of E_F and G_F measured on each individual strip by the methods described in (I). Values of E_F were measured over a length of 27·5 cm., while G_F was measured over a length of 5 cm., thus minimizing the errors discussed in § 5 of the present paper.

After these measurements were finished, all the strips were glued up so as to give at each of the above grain angles one, and in some cases two, composite 2-ply strips, the component strips having grain angles $+\theta$ and $-\theta$, and also one composite 3-ply strip, the component strips having angles $+\theta$, $-(90-\theta)$ and $+\theta$.

The glue used was Beetle W (urea-formaldehyde) with a slow hardener, and the composite strips were kept under pressure in the cold until the glue had set. They were then again conditioned to 60 % relative humidity, and E_b and G_t measured on them by the same methods and over the same lengths as in the original measurements.

The values of E_b and G_t measured on the 3-ply strips are those appropriate to a 3-ply wood of standard assembly, and having all the plies the same thickness, and cut so that the grain in the outer plies lies at an angle θ with the length of the strip. Further, according to the argument given earlier in this section, the Young's moduli and rigidity moduli measured on the 2-ply strips should approximate to E_P and G_P, and, at grain angles other than 0° and 90°, should be higher than those measured on the original simple strips.

§8. RESULTS

The results of the experimental determinations, and of calculations based on them, are given in table 1 (A) and (B).

Table 1. (Unit = 10^{10} dynes/cm.^{2})

Grain angle (degrees)	0	15	22·5	30	45	60	67·5	75	90
(A) 1- and 2-ply—									
E_F (1-ply)	14·7	6·9	4·78	3·02	1·30	1·18	1·17	0·78	0·67
E_P (2-ply)	12·8	7·6	6·3	4·05	2·25	1·64	1·52	0·97	0·87
$1-\epsilon$ $(=E_F/E_P)$	1·13	0·91	0·76	0·75	0·58	0·73	0·77	0·80	0·77
G_F (1-ply)	1·24	1·15	1·17	1·06	0·88	1·08	1·11	0·94	0·95
G_P (2-ply)	1·15	1·39	1·60	1·57	1·53	1·69	1·66	1·15	1·03
$1-\epsilon$ $(=G_F/G_P)$	1·07	0·83	0·73	0·67	0·58	0·63	0·67	0·82	0·92
(B) 3-ply—									
E_b (experimental)	12·8	6·2	5·2	3·5	2·05	2·02	1·95	1·60	1·50
E_b (calc. from E_P in A)	12·3	7·3	6·1	4·0	2·25	1·73	1·71	1·23	1·34
E_b (calc. from E_F in A)	14·0	6·6	4·6	2·9	1·30	1·25	1·31	1·03	1·22
G_t (experimental)	0·85	1·09	1·16	1·24	1·25	1·42	1·35	1·19	1·03
G_t (calc. from G_P in A)	1·14	1·39	1·61	1·58	1·53	1·68	1·66	1·17	1·04
G_t (calc. from G_F in A)	1·23	1·15	1·16	1·06	0·88	1·07	1·11	0·94	0·96

The calculated values of E_b and G_t for the 3-ply are obtained from the equations

$$E_b = 0{\cdot}04E_1 + 0{\cdot}96E_2,$$
$$G_t = 0{\cdot}04G_1 + 0{\cdot}96E_2,$$

to which (12) and (13) reduce on inserting $t_2 = t_n = 3t_1$.

Table 2 gives the theoretical values of the constants at the various grain angles, for comparison with the data in table 1 (A).

Table 2. Theoretical values—birch (unit = 10^{10} dynes/cm.^{2})

Grain angle	15	22·5	30	45	60	67·5	75
E_F $(=1/s'_{11})$	8·3	5·3	3·48	1·72	1·05	0·88	0·79
E_P $(=E_F/(1-\epsilon))$	12·0	9·3	6·8	3·6	1·73	1·29	0·94
G_F $(=1/s'_{66})$	0·94	0·81	0·72	0·65	0·72	0·81	0·94
G_P $(=G_F/(1-\epsilon))$	1·36	1·42	1·42	1·35	1·18	1·18	1·10
$1-\epsilon$	0·69	0·57	0·51	0·48	0·61	0·68	0·84

All the figures in table 2 are the result of pure calculation. They are based on equations (1) and (2) and the following fundamental coefficients (cm.^{2}/dyne $\times 10^{-10}$):—

$$\left.\begin{aligned} s_{11} = 1/E_L = 0{\cdot}0704\,; \quad s_{22} = 1/E_T = 1{\cdot}40\,; \\ 2\,s_{12} = -2\sigma_{LT}/E_L = -0{\cdot}0704\,; \quad s_{66} = 1/G_{LT} = 0{\cdot}923. \end{aligned}\right\} \quad \ldots\ldots(14)$$

The values of E_L, G_{LT} and E_T used in (14) are the averages of all the appropriate measurements made during the course of the experiments. σ_{LT} was assumed to have the value 0·5.

§ 9. DISCUSSION OF RESULTS

It has been established (see, e.g., Egner, 1937) that the Young's modulus of plywood may be higher than that calculated from the normal Young's modulus of the individual layers. It is suggested in § 6 that the origin of these increases is to be found in the difference between the pure and the free Young's moduli of the wood. It is, however, first necessary to show that the increases cannot be explained purely in terms of the Young's modulus of the glue layer. This can be done by regarding an N-ply wood as a $(2N-1)$-layered material, N of which are wood and $(N-1)$ glue. Application of equations (11) and (12) to such a plywood of fixed thickness, in which the glue layers are assumed (1) vanishingly thin, and (2) of normal thickness, allows the effect of the Young's modulus of the glue layer to be assessed (Hearmon, 1939). The calculations are not reproduced here, but the difference between the two cases is never greater than 2 %, an indication that the increases are due to the elastic properties of the wood rather than to those of the glue.

It may be noted that for a normal plywood at 0° and 90° there should theoretically be no increase, since for these grain angles, $s'_{16}=0$, and $E_F=E_P$; $G_F=G_P$. Wood, however, shows deviations from the ideal rhombic material; in particular the grain direction is not strictly uniform. It is thus possible for s'_{16} to differ from zero at 0° and 90°, in which case the constants of the plywood at these two angles might be higher than those calculated from the free constants of the individual layers.

This effect may provide a partial explanation of the fact that the experimental figures at 0° and 90° (table 1) appear to be less consistent than those at the other angles. A further reason for the inconsistency may lie in the fact that at a nominal θ of 0° or 90° the measured quantities differ from the true ones in the same sense, irrespective of whether the actual θ is greater or less than the nominal θ. At most other angles, the measured quantities differ from the true ones in opposite senses, according as the actual θ is greater or less than the nominal θ. Thus, there is a tendency for errors due to deviation of grain angle, whether local or over the whole of the strip, to be self-cancelling at most angles other than 0° and 90°, whereas at these two angles they are not self-cancelling.

Considering first the results of table 1 (A), the predicted effects are observed in almost every case. The only serious discrepancy is in the Young's modulus figures at 90°, at which angle E_F is some 20 % less than E_P, though in theory the two should be equal. The good agreement between $1-\epsilon$ determined from Young's-modulus measurements, and completely independently from rigidity-modulus measurements, should also be noticed. The general trend of all the results in this table is the same as that of the corresponding quantities in table 2, but in some cases there is a fairly large numerical disagreement. Nevertheless, there is no discrepancy large enough to necessitate modification of the conclusion arrived at in (I), namely, that the conception of wood as a rhombic system gives a good first approximation to its elastic behaviour.

Turning to table 1 (B), the majority of the observed values of E_b and G_t lie between those calculated from the experimental pure and free constants. This confirms the expectation that in the elastic distortion of plywood the conditions are a compromise between those corresponding to pure and free elastic distortion of the individual layers.

§ 10. EXPERIMENTAL RESULTS IN COMMERCIAL PLYWOODS

In order to show that the conclusion reached in the previous section is of more than academic interest, some experiments have been carried out on commercial birch plywoods. One sheet of 3-ply, $t_1 = 0{\cdot}103$; $t_2 = 0{\cdot}296$ cm., and one of 5-ply, $t_1 = 0{\cdot}047$; $t_2 = 0{\cdot}190$; $t_3 = 0{\cdot}305$ cm. (equations (11), (12) and (13)), were cut into strips in such a way that specimens were obtained with the grain angle in the outer plies 0°, 15°, 30°, 45°, 60°, 75° and 90°. At least two strips, approximately 60 cm. long by 2·5 cm. wide, were available for each plywood at all the above angles. These strips were conditioned to 60 % relative humidity before measuring the elastic constants. The methods described

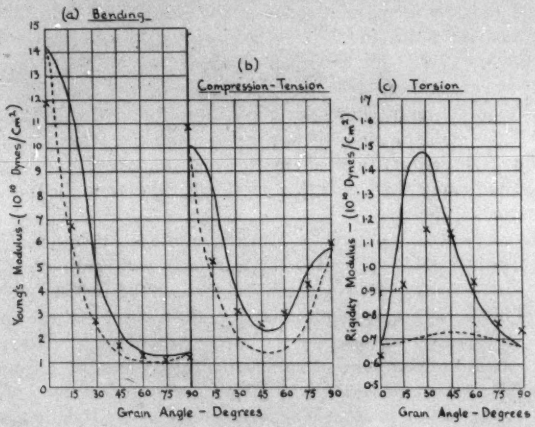

Figure 3. Birch 3-ply.

——— =theoretical pure; × =experimental; - - - - - - =theoretical free.

in (I) were again used for E_b and G_t, while E_c was measured by the familiar Kundt's-tube method (Worsnop and Flint, 1927, p. 415). Some difficulty was experienced in getting this method to work, but careful attention to experimental detail, particularly to keeping the inside of the tube and the lycopodium powder perfectly dry, enabled results to be obtained.

Figures 3 and 4 compare the experimental results with theoretical values, based on figure 1 and equations (11) to (13). The experimental results in figures 3 and 4 were obtained on strips cut from plywood which was in the assembled state when received. As no corresponding unassembled veneers were available, the fundamental constants of the individual plies could not be

determined directly, and the assumed values on which figure 1 is based were therefore used in the calculation of the theoretical points in figures 3 and 4. For this reason, comparison between theory and experiment in figures 3 and 4 is subject to considerable error, and cannot be more than qualitative.

In the case of bending, the evidence is somewhat uncertain, but as the figures stand, the tendency seems for the bending to be free rather than pure, although one or two of the experimental points lie between the two theoretical curves. In compression-tension, on the other hand, the strains for most angles seem to be more nearly pure, although at high and low grain angles the points are, on the whole, closer to the free curve.

The strongest evidence, however, comes from the torsion results. In this

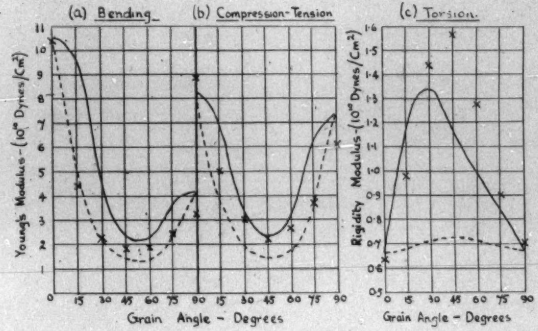

Figure 4. Birch 3-ply.

——— =theoretical pure; × =experimental; - - - - - - =theoretical free.

case there is a considerable difference between the free curve and the experimental points, but when the pure curve is taken as the basis of comparison, the discrepancy is considerably reduced.

From the point of view of wood technology it is of importance to have established this conclusion. For some purposes, when high resistance to shear and torsion is required, plywood cut at 45° is often used. The results given above show that, at least as far as the approximately elastic range is concerned, any complete explanation of this practice would have to take into account the consequences of the bending-torsion and the shear-extension coupling inherent in wood cut at an angle to the grain.

§ 11. ACKNOWLEDGEMENTS

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